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Vol. 213 FEBRUARY, 1932 No. 2

A VACUUM TECHNIC FOR THE CHEMIST.*

BY

K. HICKMAN, Ph.D.

The number of molecules in a cubic centimeter of air at the earth's surface is about 2×10^{19}. This degree of molecular crowding is a permanent quality of our atmosphere and is evidently a convenient standard of pressure; it is in multiples or fractions of the *atmosphere* that pressures are generally expressed. Our standard of emptiness, the vacuum, is not nearly as well defined. It should correspond with the complete absence of matter, or, in metric notation, with zero molecules per cubic centimeter. Certain regions between the stars contain so little material that not more than one cubic centimeter in 10 is likely to contain a molecule. An absolute vacuum might be preserved by erecting walls round some volume of interstellar space which did not at that moment contain a molecule.

Apart from the impossibility of conducting operations in outer space, all our building materials have been subjected since the beginnings of time to a gaseous environment and are so saturated with volatiles that they continue to emit molecules for a long time when placed in comparative emptiness. The walls designed to exclude stray molecules would be the first agents to introduce molecules.

* Communication No. 489 from the Kodak Research Laboratories. Presented at the stated meeting held Wednesday, February 18, 1931.

(Note.—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

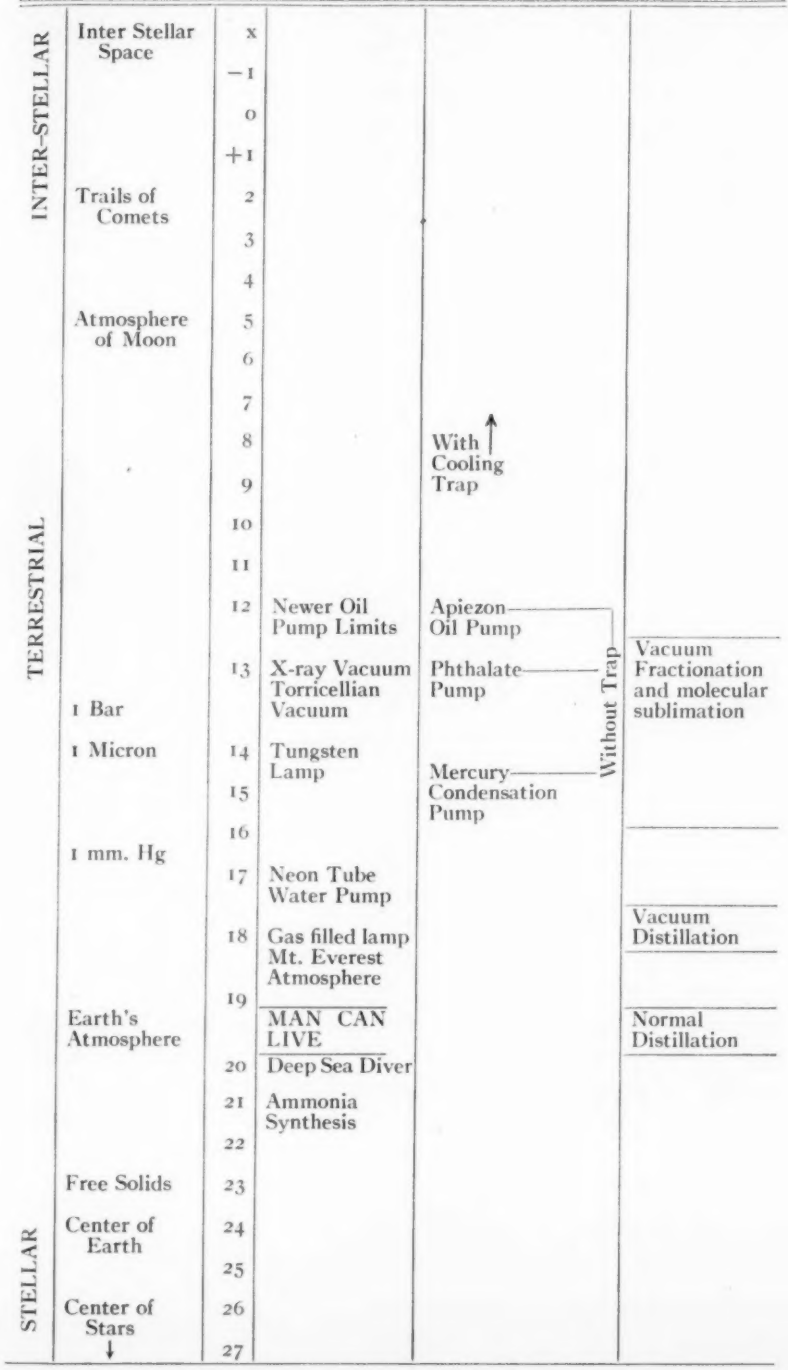
Terrestrially speaking, a vacuum is simply a reduction of pressure and a *good vacuum* is considered by each operator to be the particular degree of reduction which is good enough for his purpose. A chemical engineer may consider he has a vacuum when the original pressure has been reduced one hundred times from 10^{19} to 10^{17} molecules. The oil technologist perseveres to 10^{16} molecules, and the x-ray or vacuum spectrograph worker feels he has a really high vacuum when there are only a paltry million million molecules left in each cubic centimeter. The greatest reduction of pressure yet secured has been to 10^{8} m.p. cm.^{3} and here every device known to the physicist was invoked.[1]

Those uninitiated into the mysteries of vacuum technic will infer that it is extremely difficult to secure great reductions in pressure and equally difficult to prevent leakage or return of the molecules removed. Procedure which may be adequate for a limited reduction may be useless to secure a really high vacuum and the drastic precautions which are imperative for the latter may be employed mistakenly for the former with much unnecessary inconvenience. It is the purpose of this article to outline procedure valuable in chemistry and chemical industry.

Since there is no absolute standard either of pressure or emptiness, it is instructive to arrange certain well known phenomena in an exponential pressure, or rather molecular crowding, scale. For pressures greater than atmospheric, a pressure scale does not correspond with a molecular scale, but for estimating degrees of vacuum, the two are essentially the same.

On the molecular scale, it is seen that our terrestrial operations are crowded in a region somewhat nearer the upper than the lower limit. The animal body remains alive in a region so narrow that it is marked by a mere point on the scale; heat engines (steam or gasoline), and refrigerator compressors rise scarcely a couple of magnitudes above this point and the chemical engineer drops about the same distance below the atmospheric mark. Only the physicist has secured anything approaching a vacuum and he has much further to go before the properties of really empty space can be studied in the laboratory.

CONCENTRATION DIAGRAM OF PARTICLES OF MATTER PER CM.^{3} OF SPACE (EXPRESSED IN TERMS OF 10^x).



Although the properties of a gas undergo continuous modification with changes in pressure, the apparent behavior of a gas and our subjective attitude towards it change profoundly when rarefaction is involved instead of compression. When we pump we command, and the molecule must obey; when we suck, we beckon, but the molecule responds only at its own sweet will. An advancing piston pushes all before it, doing work in proportion to the opposing pressure and the distance traversed; a receding piston can be followed only if its speed is less than that of the molecules at the temperature prevailing. The delivery of a gas can be increased by increasing the pump speed, but the exhaustion of a system cannot be altered one whit by increasing pumping beyond a critical limit. In apparatus under pressure, a gas may be transferred from one part to another at any desired speed by regulating the pressure differences between the two parts; by increased or diminished pumping. The mean free path is, for practical purposes, infinitesimal, and although only a few molecules actually meet the piston face, all the others do so virtually by secondary bombardment; they are all pushed by the piston. In apparatus containing rarefied gas, the pressure available for transferring material from one position to another is correspondingly small. All that a pump can do is to clear the way and invite molecules to enter the emptier regions. When the free path is greater than the container, no secondary push will be received, and the molecule will wander round by a haphazard route until it strikes the orifice connecting the two regions. The only control we have in conveying rarefied gas is by adjusting the *temperature* and the *lengths*, *shapes*, and *cross sections of the pipes*.

The foregoing remarks apply to an uncondensible gas. Gas properties are not a continuous function of gas pressure at the point when the gas becomes a fluid. Most chemical vacuum problems involve a mixture of condensible gases (most generally, water or solvents) and permanent gases (air). We may distinguish two broad classes of operation:

(*a*) The permanent gas is the carrier for the vapor.
(*b*) The vapor is the carrier for the permanent gas.

In class (*a*) fall all drying operations, generally at atmospheric pressure, aided by a draught. Class (*b*) comprises vacuum operations when a cooled condenser is placed between the vessel and the pump. How the vapor carries the gas will presently appear.

APPLICATIONS OF VACUUM IN CHEMISTRY.

Low temperature distillation is usually undertaken to prevent decomposition of unstable liquids. The boiling point of a substance alters so that the logarithm of the vapor pressure varies as the inverse of the absolute temperature. The vapor pressure (boiling pressure) of water decreases from 760 mm. at 100° C. to 526 mm. at 90, to 4.6 mm. at 0° C., and to 0.034 mm. for ice at − 50°. A number of substances normally boiling with decomposition at about 350° C. at normal pressure distill at 160°–210° at 10 mm., 100–130° at .01 mm., and at 40–60° at .0001 mm. Ethylene glycol of normal B.P. 200° C. boils at 80° at 10 mm. and at 20° at 0.1 mm. pressure. A light lubricating oil with a boiling point ranging from 160° C. to 230° C. may exert 760 mm. of vapor in a heavy crude oil only when the mixture is heated above 360° C. Distillation at reduced pressure can be performed and the oil brought over at a low temperature without spoiling the heavy residue. Glycerin decomposes into acrolein and other products when boiled at normal pressure, and the pure commercial product is secured by vacuum distillation; and so on. The many kinds of commercial distillation apparatus functioning at 30 mm. − 1 mm. Hg can be judged from the advertising pages of the industrial chemical journals.

VACUUM DISTILLATION IN THE LABORATORY.

The homogeneous mixtures usually concerned may evolve a vapor of composition different from that of the parent liquid.[2] When this happens, distillation will effect a partial separation, giving a distillate richer in the lighter component and a residue richer in the heavier. The distillate and residue may be again distilled, the light fraction from the initial heavy portion and the heavy fraction from the light being combined and again distilled, and so forth, in a manner well understood. Instead of multiple distillation, a fractionating

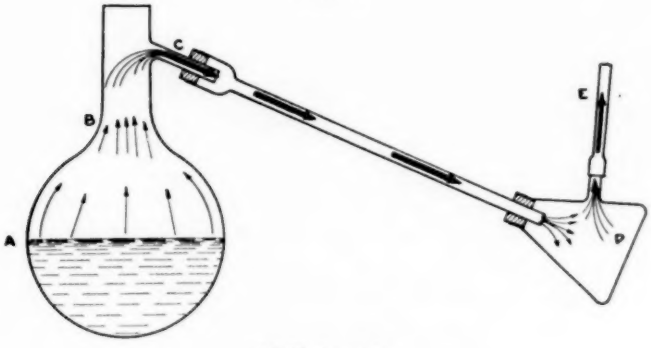
column may be employed, which subjects the condensate to many partial reëvaporations and a very perfect separation of the components will result.

The degree of vacuum under which it is possible to conduct a simple distillation is, in general, much higher than is possible for fractionation.

DISTILLATION.

A simple distillation apparatus consists of a flask, a condenser, a receiver and a connection to pump and manometer (Fig. 1). In practice, this may include the "Perkin triangle" or the "Thorne triangle" for the more convenient separation of the fractions (Fig. 2).

FIG. 1.



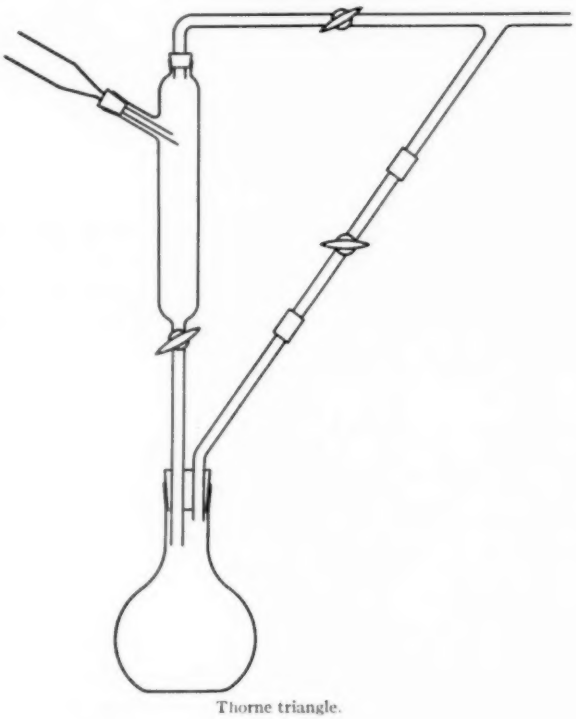
Distillation train.

Consider the apparatus of Fig. 1. The liquid in the flask is heated and gives off vapor at its surface which, at pressures above one millimeter, may be considered, for the purposes of argument, homogeneous, i.e., non-molecular. The vapor drifts in a slow stream towards the neck where it must accelerate to pass through the narrow area on its way to the still narrower side arm connecting with the condenser. The velocity in the side arm and condenser is considerable. If the liquid surface is 10 cm. in diameter and produces gas in a column moving upwards at 10 cm. per second, the velocity in the 2 cm. neck must be 25e cm. per secs. and in the side tube of diameter 5 mm., 40 meters per sec. We will suppose an adequate suction means producing substantially zero

pressure in the receiver. At what pressure must the gas be generated in the flask to force its way out at 40 meters a second through the narrow side arm?

The weight of gas, p.v., transferred depends[3] on the pressure differences at the two ends of the pipe, in this case p,

FIG. 2.



Thorne triangle.

on the fourth power of the tube diameter, the length of the tube, and on the viscosity of the gas. Thus:

$$\text{p.v.} = \frac{\pi D^4}{128\eta L} \cdot p^2,$$

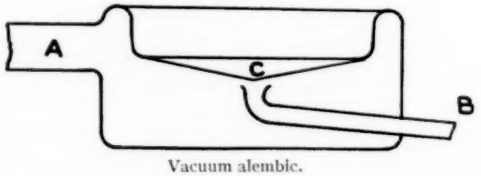
where $v = 780$ cms.^{3} per sec., L is 20 cm. and $\eta = 1.7 \times 10^{-4}$.

$$p = \frac{780 \times 128 \times 1.7 \times 10^4 \times 20 \times 750}{(0.5)^4 \times 10^6} = 4.07 \text{ mm.}$$

This leads to the interesting conclusion that no matter what low degree of pressure a manometer attached to the receiver may read, the real pressure in the flask when vapor is delivered at this rate will be *above 4 mm.* This would correspond with a distillation rate of one drop every 2–8 seconds for organic substances averaging 150 in molecular weight—not an unduly vigorous yield. This discrepancy between real and apparent pressure, intuitively feared but not consciously recognized, has forced chemists to perform most low pressure distillations at pressures above 10 mm. This is a pity because there is often as much to be gained by further reduction as has been secured at 10 mm. pressure.

The disabilities in the way of low pressure manipulation can be avoided if we eliminate the neck and side arm [4] of the still and place the condensing surface close to the liquid. This has been accomplished in the evaporative still of Burch [5] and the molecular still of Washburn.[6] Such apparatus is not convenient for every day use by the chemist and it is advisable to design a simple glass evaporative still.

FIG. 3.



Vacuum alembic.

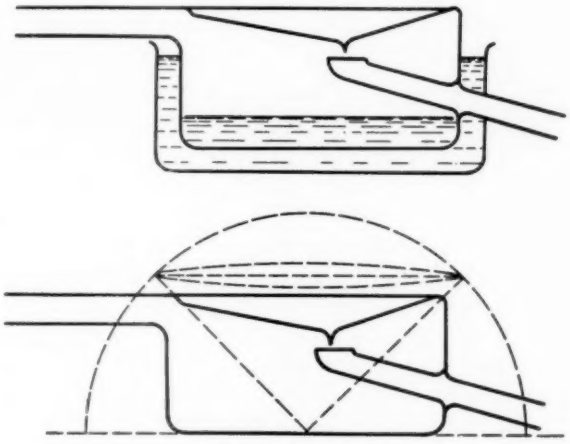
The alembic featured in Fig. 3 enables distillation to be performed at a fraction of a micron as rapidly as the ordinary flask will permit at a fraction of a centimeter of mercury. Under a sufficiently high vacuum molecules leave the hot surface of the liquid and condense on the concave ceiling whence the drops fall into the collecting arm and pass to the receiver. The rate of distillation will be proportional to:

(1) The temperature of the evaporating surface. This may differ materially from the temperatures of the bulk of liquid, the difference being a function of the viscosity and the coefficient of expansion. Since there is no ebullition there is no stirring other than that induced by convection currents.

(2) The coefficient of condensation. Until the condensing surface has been wetted by distillate the coefficient may be very small. With temperature differences of only 15–30° C. between evaporating surface and ceiling, it may be necessary to start condensation by applying solid CO_2 or liquid air. The coefficient of condensation may be unity or near unity for a wetted surface, and drastic cooling produces no appreciable increase in yield.

(3) The solid angle subtended by the condensing surface. When the entire vessel is jacketed this is nearly 180° (Fig. 4A). With the simpler arrangement of Fig. 4B, it is the area cut by the cone shown in dotted lines.

FIGS. 4A AND 4B.



Vacuum alembic.

(4) The buffering effect of residual gas. The gas may be air, volatiles, or products of cracking. The first may be removed easily, but the second, which are generally present can only be removed by that essential combination—adequate pumping and a wide delivery tube.

Let the delivery tube be 1.5 cm. in diameter and let it be connected with a trap and pump, the total admittance of which is equivalent to a circular opening .8 cm. in diameter. Let the diameter of the ceiling be 10 cm. The chance that an evaporated molecule will be removed by exit through the

delivery tube or by condensation on the ceiling will be $.8^2 : 10^2$ or, say, 1 : 156. The phlegmatic material being distilled will condense 156 parts on the ceiling and one part will be lost in the delivery tube. The volatiles which do not condense can only leave by the side tube once in every 156 collisions. In general, they will be of lower molecular weight and more than one volatile molecule may be produced by the cracking of one phlegmatic molecule. It is safe to say that a decomposition of one-half of 1 per cent. of the still's contents will result in the retention of as many more uncondensible molecules as there are condensible in the still at any moment.

Now, the mean free path [7] of the molecules in the vessel is a composite of their average molecular weight, temperature, density, and viscosity. Thus: $\eta = .3502\ \rho 14551\ L\sqrt{T/M}$; which yields a figure of about 10 cm. for air at room temperature and one micron pressure.

A phlegmatic liquid distilling at 1 micron would yield molecules having less than a 50 per cent. chance of collision with air molecules if the residual air pressure was also 1 micron in apparatus of the size we are considering, and distillation would hardly be hindered. If the residual air, volatiles and cracking products reached .01 mm., the rate of distillation would be cut ten times. One-tenth of a millimeter residuals (a very ordinary concentration and one not easily detectable with the McLeod gauge) would practically stop distillation.

For convenience we repeat, with the caution that it assumes a condensation coefficient of unity, Langmuir's equation expressing the rate of distillation per sq. cm. superficial area.

$$W = 4.374 \times 10^{-5} \times 60P\sqrt{\frac{M}{T}},$$

where W is the weight in grams of a substance of mol. wt. M at P bars (1 bar = 1.76 microns = .00076 mm.) and an absolute temperature T. The temperature of the liquid in the still equals the temperature T of the fluid surface only at very slow rates of distillation.

FRACTIONATION.

Separations in a single operation designed to achieve more than one theoretical "plate" involve washing the vapors with

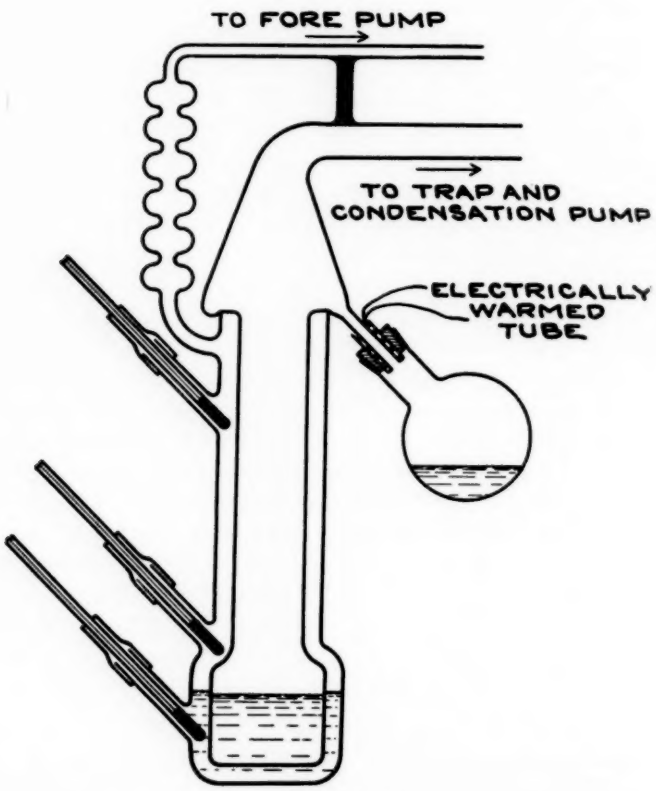
intermediate portions of distillate. The well known *fractionating column* allows this washing to occur by leading the vapor through or round the condensate by a tortuous path along which the vapor suffers a substantial fall in pressure. The argument in our opening paragraphs emphasized that this fall of pressure is just what a true vacuum distillation should avoid, otherwise the whole object in employing a vacuum is frustrated.[8]

Fortunately, under high vacuum the tortuous path is not necessary. The rapidity with which equilibrium is established between the vapor and liquid is directly proportional to the free path of the vapor molecules. At atmospheric pressure the free path is of the order of .0001 mm. and long contact and violent swirling are necessary to bring all the gas molecules to the liquid surface. Similarly, the liquid must be in violent agitation because of the correspondingly smaller free path within its own boundaries. At one millimeter absolute pressure, the fractionating column need present only 1/760th the complexity of path, and at pressures of 10 microns or less a straight tube affords at once an excellent fractionating column and a trifling barrier to the gas. Consider, for instance, a column 2 cm. radius, 25 cm. long with a boiler at one end and a condenser at the other (Fig. 5). The mean free path of a certain vapor will be assumed to be 5 cm. at a pressure of 1 micron measured at the point A, and the pressure of residual gas will be supposed to be negligible. A molecule of vapor (A) will have at least a 90 per cent. chance of meeting a wall without striking a sister molecule, and less than a 1 per cent. chance, calculated as the ratio of the collecting areas involved, of reaching the condensing end. Even for a molecule projected directly upwards, the chance of surviving five times the mean path is only one in 148 * so that the open end offers no real means of escape at the first projection. One-half of the molecules should be collected by the first 1.2 cm. of tube surface, and of those that reevaporate in an upward direction, half will collide with the next 1.2 cm. of wall.

* This figure is taken from Jeans, "Dynamic Theory of Gases," Cambridge University Press, 1904, p. 236.

We are not prepared to develop the expression for the average number of condensations and reëvaporations a molecule will experience in its passage through the tube, but manifestly it will be a substantial number. Now, at one micron a collective area of 2 cm. at the base will not admit more

FIG. 5.



Fractionating alembic.

than .04 g. per sec. of a substance of mol. wt. 200, a quantity totally insufficient to warm the walls of the tube. In fact, experience shows that all but the first centimeter or so will remain quite cold even when carefully lagged, and none of the molecules will reach the true condenser at the top.

The difficulties in the way of a progressive electrical warming need only be experienced for such a method to be discarded. The correct rate of decay of temperature as one ascends the tube must be guessed and also the right general temperature level. The windings must be spaced with progressively greater gaps and an opaque lagging is not desirable since the condition of the inner walls cannot then be watched.

Again, the difficulty contains the germ of its own cure. Theoretically, in a perfect vacuum a liquid will emit vapor at any temperature. If the liquid is at one temperature and parts of the vessel are at another lower temperature, the liquid may be said to boil and condense on the colder portion irrespective of the absolute temperatures concerned. Only at low temperatures the quantities involved are so small that no detectable quantity would be transferred during a long experiment. There is, therefore, a certain temperature region, typical of each liquid, at which vaporization in a particular piece of apparatus becomes perceptible. Thus, a phlegmatic liquid may be heated in the end of a long tube under vacuum. A certain temperature must be reached before the faintest mist of condensate will appear on the cold upper walls in half an hour. At a little higher temperature active distillation is apparent, the evaporating liquid surface appears disturbed and the tube becomes hot where rapid condensation is occurring. When butyl phthalate is heated in a vacuum of less than 1 micron, the first evaporative temperature is about 100° C., corresponding to the relatively high vapor pressure of .06 mm. Active distillation begins at 130° C. and vigorous boiling with refluxing at 150° C., corresponding to pressures of from .4 to .8 mm. These phenomena are repeatable and easily recognized and they indicate the pressures necessary to force vapor in these quantities up a tube about 1 cm. in diameter. With chlornaphthalene the events may be observed at 30° C., 50° C. and 70° C. respectively; with tricresyl phosphate, at 150°, 170° and 180° C. Each phlegmatic liquid has its own characteristic temperature for apparent boiling in a vacuum.

The phenomena can be utilized in vacuum fractionation to surround the fractionating tube with a jacket in which a

phlegmatic liquid appropriate to the experiment is boiled under high vacuum.*

A convenient apparatus is shown in Fig. 5 and its use will be described in the distillation, for example, of cocoanut oil.

A heavy crank case lubricating oil is degassed and distilled in the alembic of Fig. 3, and the portion coming over at a rate of one drop a second between 160–180° C. is collected. This is placed in the outer jacket to the level shown. The cocoanut oil is placed in the inner container and the pipe V connected with the pump and exhausted while gentle heat is applied. When frothing is over and the pressure of residual air has fallen below one micron, the heat is increased till visible refluxing is taking place in the outer jacket. The three thermometers, reading from the lower-most, will read approximately 195, 183, 165, and *all* the excess heater oil vapor should condense in the lower part of the safety spiral. The cocoanut oil will evolve a small fraction of yellowish oil, a second fraction of oil which will just solidify on standing, and a sharply cut fraction of oil setting to a white crystallization wax. The distillation of 30 grams will take perhaps two days, but the apparatus may be left overnight without attention.

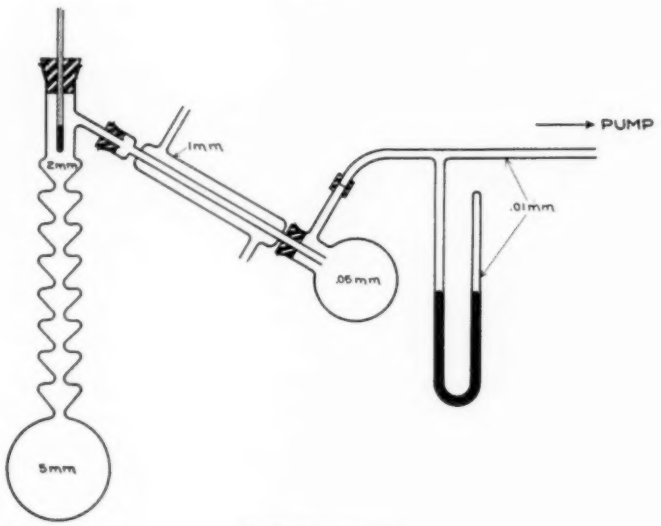
In the previous examples, we arrange the temperature, eliminate the residual gases and then assume or calculate the pressure of the vapor under distillation. The vapor pressure is directional and is not amenable to measurement with existing gauges. The constituents of a phlegmatic mixture can be *separated*, but they cannot be *identified* by the temperature-pressure relation at these low pressures.

Accurate temperature-pressure relations can be determined between .1 mm. and the conventional 10 mm. by the device of the automanometer.[8] Consider again the schematic distillation train such as shown in Fig. 6. We will ascribe likely pressures to the various vapor regions during a distillation and mark them in the figure. We have the simple requirement that the vapor pressure must be read at the thermometer

* The term "high vacuum" used in this sense has a specialized meaning. It means a pressure of residual air so much less than the pressure of the vapor above the liquid that vapor is generated entirely without interference. The exact degree of vacuum will be immaterial.

bulb, and the thermometer bulb must be placed where the vapor will receive no further refluxing before condensation. No ordinary gauge can read the vapor pressure since the very act of conveying the vapor to the gauge would result in its condensation or expansion. Accordingly, we fit the thermometer chamber with a short U-tube which will fill with the hot condensate. The farther end we connect with a source of known pressure or of high vacuum. The sum of the automanometer readings and the external manometer reading

FIG. 6.



Distillation apparatus.

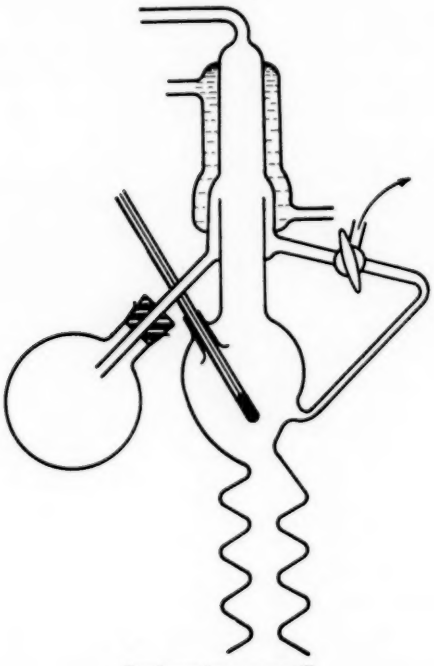
then gives the true pressure in the thermometer cavity with an accuracy of perhaps one-fiftieth of a millimeter of Hg. It is, of course, necessary to know the specific gravity of the liquid condensing at the moment of reading the pressure.

Two types of flask have been evolved, the simpler cooperating with an external gauge, the other independent of gauges. The independent type relies on the substantially perfect vacuum produced when a phlegmatic vapor is conveyed past a discontinuity. The still head of Fig. 7 has a condensation pump placed above the thermometer cavity and

the condensate runs into the receiving flask at the side. The differing capillary actions at the hot and cold ends of the manometer require that the zero point be found empirically by momentarily relieving the vacuum, so that the pressure may equalize in the two limbs.

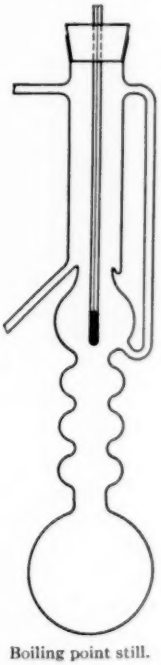
The condensation still can act satisfactorily only with liquids having negligible vapor pressures at room temperature.

FIG. 7.



Condensation pump still.

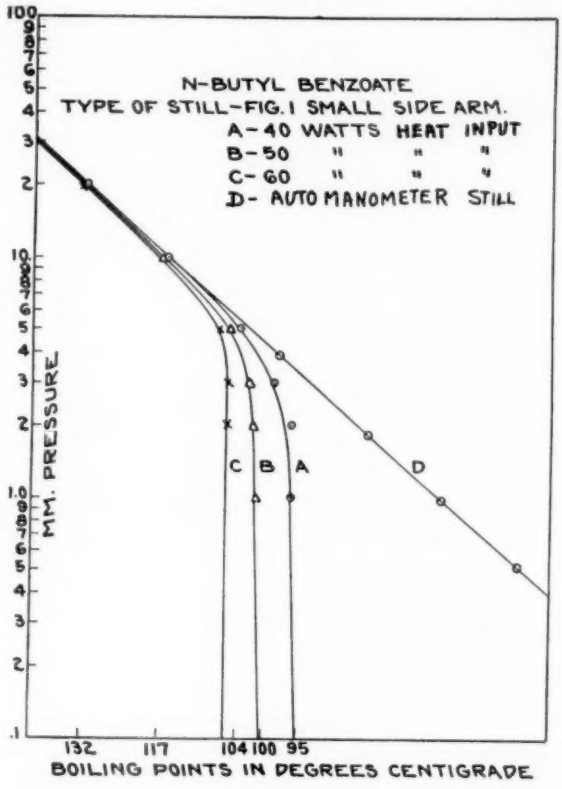
FIG. 8.



Boiling point still.

In the large class of compounds with normal pressure boiling points round 200–250° C., it is imperative to operate with residual air in the apparatus. For this, the boiling point flask of Fig. 8 is convenient. The differences between the apparent and true boiling points as recorded in an ordinary distillation apparatus and in the condensation still are shown in Fig. 9. Extensive amounts of data have been recorded elsewhere.

FIG. 9.



Distillation curves.

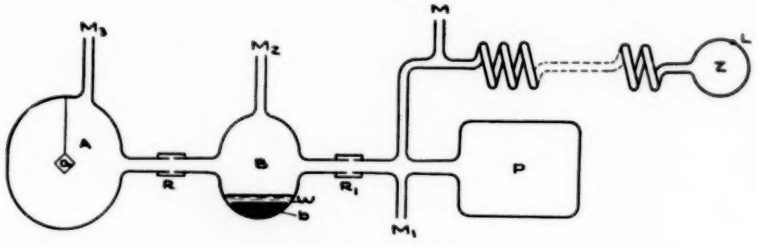
THE RETENTION AND MEASUREMENT OF LOW PRESSURES.

Consider the vessels (A) and (B), of Fig. 10, connected in series to a pump (P) and in parallel with a remote vessel (Z). (A) contains a system (a), such as material to be sublimed, a cathode to be degassed, or what not; and (B) contains a substance (b) for example mercury, covered perhaps with a little water (w). R, R' are rubber joints, L is a very small leak in (Z) and M, $M_1 \cdots$ are possible positions for a manometer connection. The pump P is supposed to have an indefinitely large capacity for producing a vacuum, and is limited in effect only by the admittance of the tube to P.

Every particle entering P is supposed to disappear. We will start the vacuum pump and watch the phenomena as the pressure decreases. At each stage we shall have to make alterations in procedure according to the *kind* as well as the *degree* of vacuum conditions required.

At the start air leaves (A), (B) and (Z) for the pump. Any water vapor in (B) which might harm (a) will be prevented from entering (A) because of the gas stream leaving (A). The contents of (B) will become cool and heat may be required to prevent a residue of ice lingering behind. Now, the walls of the vessel slowly release gas. The rubber connections R, R_1 give off quantities of vapor and the mixed material passes to the pump. The mean free path of the molecules is increasing and at a pressure of .001 mm. the

FIG. 10.



Schematic apparatus.

average uninterrupted travel of a mercury atom is about 4 cm. Mercury vapor will pass from (B) to (A) by direct bombardment of the walls *against the main stream of gas*. Considerable counter diffusion takes place at 10 times greater pressure and it has been troublesome at .05 mm. We may question what happens below 1 micron, which is below the vapor pressure of mercury at room temperature. The answer is that the pressure does not fall below 2 microns (vapor tension of Hg = 2.4 microns at 20° C.) with respect to mercury, though it may fall to unmeasurable amounts with respect to other gases. The mercury atoms are free to diffuse in every direction and will stream through the tortuous path into (Z) even against the gas which is entering through the leak (L) (always supposing the leak is insufficient to raise the pressure

above a few microns). Whenever, therefore, the pressure falls below about .01 mm. we have to decide whether there is any material which will be harmful if it diffuses *against the main vapor stream*. The materials are generally mercury vapor, hydrocarbons from rubber tubing and sealing greases, adsorbed water vapor or residual solvents, and in special cases butyl phthalate vapors. The most satisfactory way to prevent the back passage of these substances is to include a freezing trap between the vessel and the *last* glass-rubber joint.

All the substances mentioned may be dangerous to the physicist; the chemist may object to mercury, but only when performing evaporation distillations. Mercury at room temperature will reduce the mean free path to about 1 cm., whereas a saturated butyl phthalate vapor gives a free path of some 6 cm.

The procedure will differ according to whether the vacuum is to be maintained statically or dynamically. The physicist generally wishes to seal off a bulb and maintain therein a good vacuum indefinitely; the chemist requires a vacuum only during an experiment. Accordingly, most of the precautions imperative to the physicist may be omitted by the chemist. Thus, the physicist carefully bakes out his glassware, eliminates all rubber seals, and makes a drastic search for leaks. The chemist is concerned with securing an adequate pressure reduction at the time of the experiment, and if this reduction is an expression of the equilibrium between pumping and the infiltration of air, volatiles and emanations from rubber tubing and greases, it is seldom of serious moment.

The choice of a lead off place for the manometer presents no difficulties in a sealed static system free from leaks. In a system under rapid pumping it needs careful selection. We are dealing with individual projectiles rather than an elastic fluid, and there will be places generally missed by the projectiles and others which present an unfair catchment area. The position M, in Fig. 10, is bad because the leak in (Z) will give too high a reading. M_1 is very bad not only because of unfair access to the pump but because the molecules rushing by the T-joint will tend to reduce the pressure in the

pipe leading to the gauge. M_2 is the best place for measuring the pressure in (B) and M_3 in (A). With much reaction product leaving (A) there may be a fall of appreciable parts of a millimeter between the two vessels.

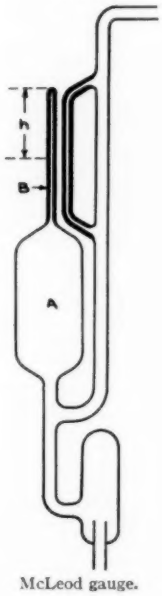
There is no universal type of manometer which may be applied to give, in a single reading, without adjustment, a true measure of the pressure within a vessel. Of the many types available, we shall touch on the properties of 5, which are listed in an order of no significance except for the development of the argument:

(1) The McLeod gauge
(2) The U-tube manometer
(3) Modified U-tube manometers
(4) Hot wire (Pirani) gauges
(5) The ionization gauge.

No physical measuring device has been more useful and more misunderstood than the compression gauge of McLeod. A known large volume of the "Vacuum" is imprisoned in (A) Fig. 11 and is compressed by a mercury plunger into a calibrated tube (B) under a final head of mercury equal to the length occupied by the gas in the measuring tube. Boyle's law is assumed for the gaseous behavior under compression, whence $P = \frac{a}{V}h^2$, where (a) is the volume of the capillary per mm. run, (V) the volume of the vessel and capillary, and (h) the height of the gas compressed under a similar head of mercury (h). It is apparent that the only pressure the gauge will record is that of the permanent gases entering (A). Volatile substances will condense partially or wholly during the compression stroke and the reading then *may* be the pressure of permanent gases in the system minus the volatiles. The all-pervading mercury vapor is eliminated but not necessarily the water vapor or such usual solvents as alcohol, acetone or ether. Water has a vapor tension of 25–30 mm. on a warm day and the entire apparatus train may be above room temperature. Readings of the McLeod gauge when h is less than 20 mm. will be influenced by water in degrees varying with the adsorption exerted by the glass, the condition of the mercury and a number of variables. Similarly, alcohol

will play an unpredictable part up to pressure of $h = 80$ mm., and acetone and ether at all usual temperatures. A freezing trap to remove volatiles *must* be employed if the gauge is to give a reading of the permanent gases. The indication given by the McLeod gauge attached to a system of mixed gases and volatiles can be found by studying the simple arrangement of Fig. 12. The pressure is to be lowered in stages to the highest attainable vacuum and it is supposed that an appropriate liquid is being distilled at each stage.

FIG. 11.

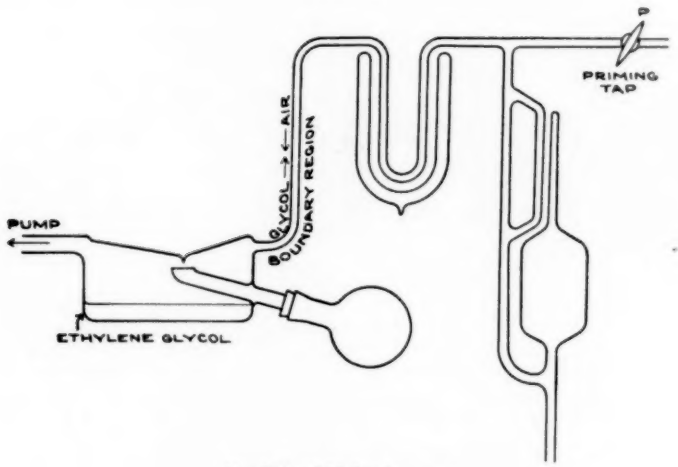


McLeod gauge.

In the first stage a liquid such as ethylene glycol can be distilled at 5–10 mm. pressure at 90–100° C. Initially, the apparatus contains air and the pressure will be reduced to 5 mm. before the pump cock is closed. The liquid is now boiled and a mixture of air and vapor streams towards the trap. After a few centimeters travel, the vapor condenses practically completely and only the air can pass on. The molecular free path averages .01 mm., which is small compared with the size of the pipe, and none of the air is able to diffuse

back against the oncoming vapor. The point of condensation of the vapor will advance or recede till the air pressure is in exact equilibrium with the vapor pressure at this point and, allowing for a negligible friction in the passage of the vapor from the still a few centimeters along the tube, the McLeod gauge will read in terms of non-condensible gas the real pressure of the condensible gases in the still. Note that in this instance the McLeod gauge is giving an accurate reading of a condensible vapor.

FIG. 12.



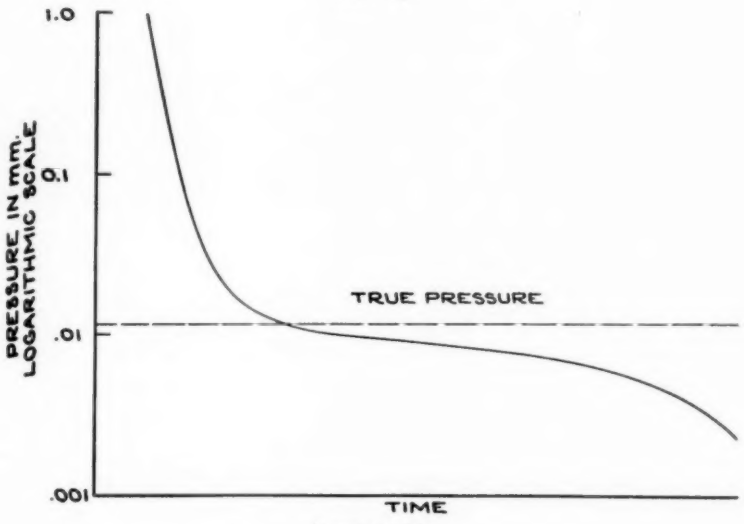
Alembic and McLeod gauge.

Stage 2.—Distillation is performed at .5–1.0 mm. Here the mean free path is about .1 mm. The gauge will read differently according to whether the exhaust pump is left operating or shut off. With operation, vapor will stream to the pump in one direction and to the gauge in the other. Very slight counter diffusion will take place and the pressure readings will have the form shown in Fig. 13. The first point of inflection is so marked and the decay of pressure thereafter so gradual that the true pressure can easily be recognized. It may be necessary to admit a little air between readings to reprime the gauge.

Stage 3.—Distillation is below .05 mm. Experimental liquid is tricresyl phosphate. The vapor and the air now

have paths which do not interfere and the McLeod gauge will give a true picture of the hindrance to distillation afforded by the blanket of residual air, but will *not* indicate the quantity of residual solvents doing the same thing. The readings tell nothing at all about the pressure of the distilling vapor except as interpreted by experience. If, from experience, the phlegmatic liquid is judged quite free from solvents or cracking products and if the pressure of residual air is below .001 mm., then the rate of condensation will yield a *computed* vapor pressure, employing Langmuir's equation and the constants

FIG. 13.



Pressure curve.

of the vessel. The writer has been criticized for recommending the McLeod gauge for use with the fractionating columns of Figs. 3, 7 and 8. He reiterates that the gauge is quite satisfactory for pressures above .1 mm. *for phlegmatic vapors* free from light solvents. Although a pressure of .1 mm. is not within the usual McLeod region, the .1 mm. should be read with an accuracy of .005 mm., which is difficult with all other gauges except perhaps those of Class 3. We consider a McLeod gauge indispensable for many degrees of vacuum distillation.

2. The U-tube manometer is generally filled with mercury and is readable, when clean, to .2 mm. with the naked eye. A few weeks in the organic laboratory will reduce its usefulness ten-fold. It has the great merit that when properly filled, its indications are absolute and need no interpretation. It is not accurate or sensitive enough for the work described in this paper.

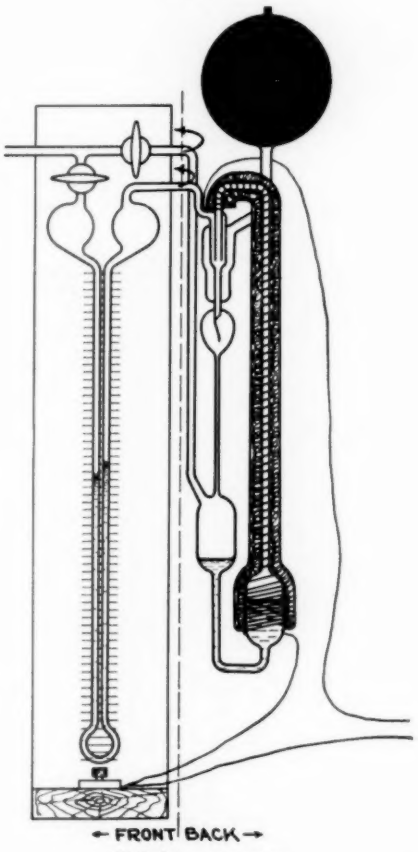
3. The modified U-tube.—The columns may be slanted or the mercury may be replaced by a lighter [8] phlegmatic liquid. An optical lever [9] floating on the mercury extends the sensitivity to the vapor pressure of the metal itself, and an external lever extends the sensitivity to .02 mm. We shall describe the U-tube filled with phlegmatic liquid for laboratory use, and the tipping arm gauge [10] for industrial recording.

The Butyl Phthalate Gauge.—All phlegmatic organic liquids dissolve air and water vapor and contain minute traces of residual solvents. Placed in a U-tube with one limb closed, application of vacuum to the open ends fails to dislodge the liquid from the closed end. When by heat or electric spark a breakaway is secured, the space is never truly vacuous and the liquid can be made to stand higher in the open limb. A gauge has accordingly been constructed which has a miniature pumping system at the closed end to secure and maintain a sufficiently good vacuum. In Fig. 14 the portion to the right is really situated at the back of the gauge. It comprises a butyl phthalate condensation pump of the inverted Langmuir type, a Sprengel fall tube to carry away gases by trapping bubbles in the condensed phthalate, and a gas storage vessel. The pump is electrically warmed and the gauge must be matured each day by direct connection with a mechanical oil pump for an hour before using. Manifestly, the device is cumbersome and is liable to accidental derangement. Sudden admission of air drives the phthalate from the manometer into the pumping portion and vice versa, necessitating recalibration, if not refilling. However, properly cared for the gauge provides a reading of the total pressure which is accurate within the working range.

This gauge should be used with the fractionating columns of Figs. 7 and 8, in which it will generally indicate all the gas pressure loading the column of vapor. It is useful for

most phlegmatic liquids in the sublimation still. There is a serious objection to any organic filling for a manometer in that vapors will dissolve in the oil, reducing the pressure above the column at one moment, increasing it the next.

FIG. 14.



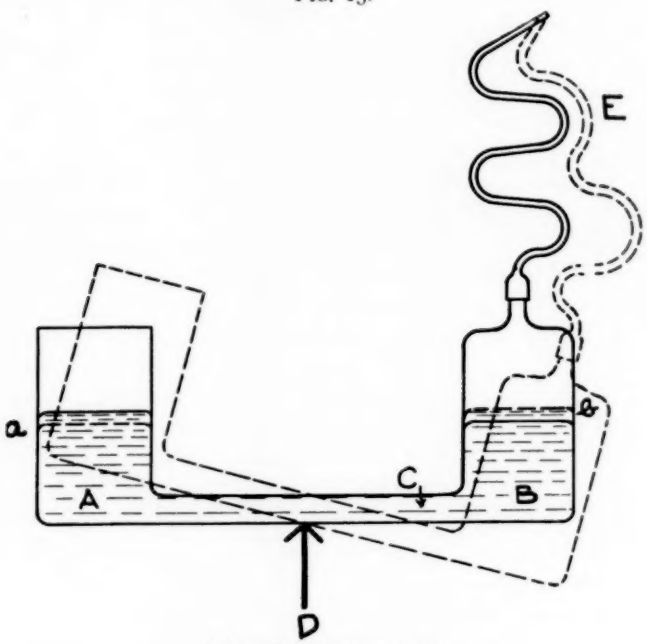
Phthalate manometer.

Practically, the effect is limited to the region of the meniscus and equilibrium is established almost immediately. Precautions: The tube from the gauge to the apparatus should be short and wide and the stop cocks of large bore, kept free from obstruction.

The tipping U-tube manometer is useful for industrial vacuum distillation and drying operations requiring pressures down to .05–.02 mm. Hg. It is useful in the laboratory for recording the rate of drying, degassing, etc., under vacuum. The principle can be understood with reference to Fig. 15.

A and *B* are the closed and working limbs, respectively, of a squat U-tube which is pivoted on the cross member at (*D*). The limbs are supposed to be filled with mercury and

FIG. 15.



Schematic tipping arm gauge.

exactly balanced with a torricellian vacuum in (*A*) and an applied good vacuum in (*B*). Let, now, a slight pressure be admitted through (*E*). Mercury will transfer from (*B*) to (*A*) until new levels shown by the dotted line (*a*), (*b*) are reached. (*A*) is now heavier than (*B*) and the system now tips down at (*A*)'s end. At the same time the levels in the two limbs, as determined by the gas pressure, are disturbed and more mercury must pass from (*B*) to (*A*) to restore the

relation. (*B*) is now heavier still and tips further, taking more mercury from (*A*) till a complete upset occurs. The system *A*, *C*, *D* is thus in unstable equilibrium and can be made to tumble over by the most trifling change in pressure. If, however, a spring or float counterpoise is attached to provide a counter turning movement, the tubes will take up a new position dependent on the relative strength of the spring. A practically useful gauge designed according to this principle is shown in Fig. 16. A 10 to 1 magnification of the

FIG. 16.

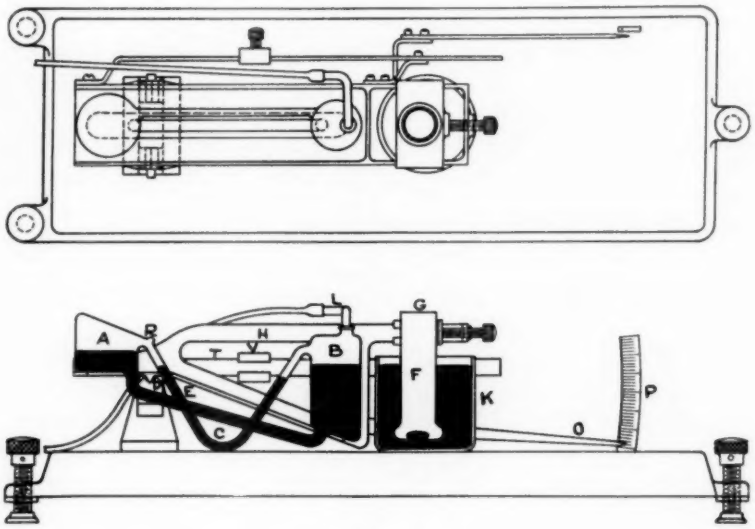


Photo tipping arm gauge.

usual U-tube readings is obtainable and a pressure change of 10 mm. will move the pointer 10 cm. on the scale or drum. A number of precautions have to be observed when using the gauge which is accurate only for changes of pressure lasting a minute or longer.

4 and 5. Electrical Gauges.—Pressures down to one micron can be measured by observing the cooling effect of the gas on an electrically heated wire which also constitutes one arm of a Wheatstone bridge. Measurements of the out-of-balance

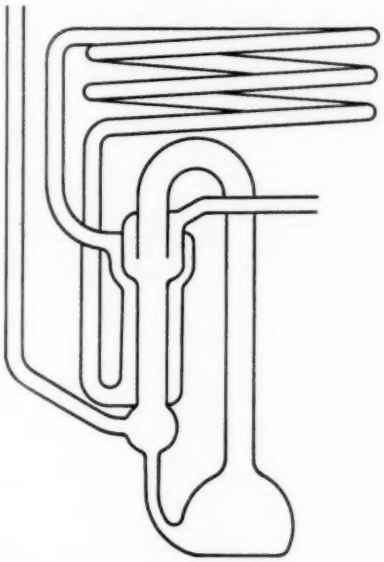
current yield measurements of pressure. This extremely simple gauge, first constructed by Pirani [11] from a tantalum filament lamp, has been improved by Hale [12] and others [13] and is available as a robust technical instrument. It will replace the *well trapped* McLeod gauge, but its behavior and the recession of zero point in the presence of volatiles is too unpredictable for it to receive our recommendation for the present use. The same qualifications, only in greater degree, apply to ionization [14] gauges designed for pressures below one micron. If the organic laboratory has induction furnace equipment for baking out the gauge after the lowest pressures have been secured, the instrument will yield a valuable indication of the pressure.

Condensation Pumps and Traps.—It is evident that any molecule wandering into a uniform stream of molecular projectiles is more likely to proceed with the stream than in any other direction. Molecules in a tube connected with such a stream will leave the tube never to return—they will have been eliminated by diffusion pumping. Uniform streams of molecular projectiles, free from laterally moving particles, are generated only by special means and at low pressures. In the well known diffusion condensation pump the lateral molecules may be eliminated first, according to Buckley,[15] or after pumping action, according to Langmuir.[16] The latter form is operable at higher jet pressures proving more powerful pumping, and anticipates the other forms in the literature.

The practical relationship of the condensation pump to the mechanical oil pump is interesting. Let the rotary plunger of an oil pump remove unit volume gas each revolution, and let the mass of the gas at atmospheric pressure be (M) and the work done by the pump in removing this quantity (W). At 1 mm. the volume removed is still unity but the mass is reduced to $\frac{M}{760}$ and the work performed to $\frac{W}{760}$. At a pressure of one bar the work done by the pump is one-millionth of the latter's initial capacity, and the mass of material conveyed is about one-millionth—other factors being equal. Where a vessel free from volatiles or occluded substances is concerned, the pump will reduce the pressure at a uniform rate nearly to its lower pressure limit, but when

appreciable quantities of materials must be transferred as in chemical vacuum work, it is essential that the volumetric efficiency of the pumping system shall increase enormously with reduction of pressure. This is achieved with the mechanical pump and condensation pump running in tandem; and the condensation pump may be regarded as a simple molecular heat engine which retains some efficiency at low pressures.

FIG. 17.



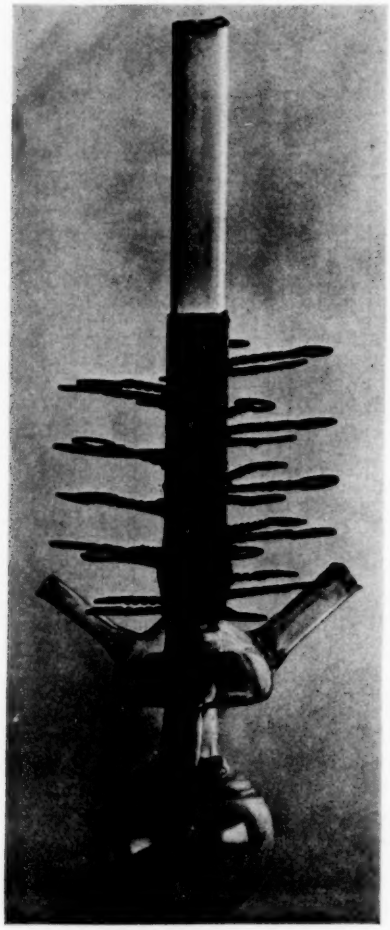
Condensation pump.

We have said that a high vacuum pumping system is established whenever a stream of molecular projectiles free from laterally moving particles passes an orifice. Any gas can, therefore, form the working fluid, but only those gases are convenient whose particles are completely condensible at laboratory working temperatures. Condensation is the most convenient means for removing lateral particles and for retaining the working fluid in a closed system.

Of all substances at present known, mercury forms the most generally useful pump filling. Its pressure at room temperature is but a few microns. A small, rotary oil pump

will evacuate to 4 microns at a uniform rate of about 100 cms. per second. The same pump backed with a mercury condensation pump of jet diameter 1.5 cm. can exhibit a pumping

FIG. 18.



Condensation pump.

speed of 5000 c.c. per second at 4 microns; below 4 microns the pump will continue to exhaust all gases except mercury. The pressure will fall with respect to these gases and with

respect to mercury if a condensing trap is placed between the pump and apparatus.

It is obvious that a trap is required for total pressures below that of mercury; is it required for pressures of, say, 10 microns? The answer lies in the part that mercury will play in the apparatus. If it contributes to the pressure *per se* and 10 microns pressure is not objectionable, no trap is necessary. If, however, the vacuum space contains a host for mercury, and such receptive surfaces are presented by the metal parts in vacuum spectrographs, by the contents of sublimation stills used for subliming metals or alloys, by the working parts of certain thermionic tubes and photocells, large and harmful masses of mercury will diffuse backwards against the pumping stream unless a trap is interposed. For the kind of work described in this paper, the criteria for the necessity of a trap are comprised by the question: Is the vapor pressure of the pump filling at room temperature above that desired in the apparatus? Will the vapor of the pump filling merely contribute to the overall pressure? Will the vapor of the pump filling react with the materials under treatment? If questions 1 and 3 are in the affirmative a trap must be used. Fortunately fillings can often be found which are inoffensive on both counts. In Langmuir's first patent aniline and water are cited as possible alternatives to mercury under suitable conditions, but workable alternatives were not forthcoming until the introduction of Apiezon oil and butyl phthalate. The vapor pressure of butyl phthalate is less than a micron at room temperature and this liquid may be used as a pump filling *without a trap* for all distillation or sublimation experiments so far described. For the vacuum spectrograph, traps cooled with solid CO_2 should be used. Apiezon oil [17] has a lower vapor pressure than butyl phthalate and affords a reliable pump filling.

When it is necessary to suck 10 c.c. of liquid into a pipette one does not select the operator with the largest lungs because the smallest human lung is adequate for the task. When high vacuum is in question it is quite usual to pay great attention to the capacity of the pump, even though some constriction analogous to the pipette orifice limits the passage of gas. A useful empirical working rule is that the jets of

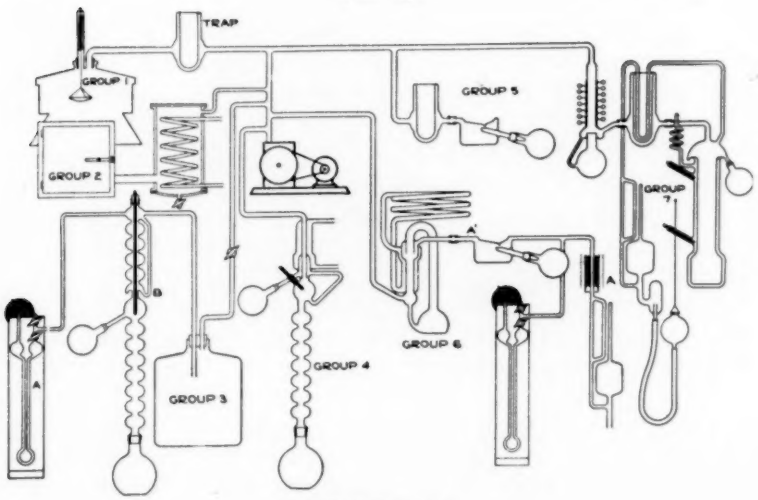
the condensation pump need be no bigger than the cross-section of the smallest portion of the tube between the pump and the apparatus.[18] For much work, this limits the useful jet size to 1 cm. diameter and such pumps are small enough for air cooling. The models illustrated in Figs. 17 and 18 are sufficient for our present work, and are independent of cooling water supply. One of these pumps has been operated continuously day and night for 5 months maintaining a vacuum beyond measurement with a 400 cm.³ McLeod gauge.

THE VACUUM ASSEMBLY.

We have felt that the recommendation of highly specialized apparatus for general routine work carries the obligation to make such apparatus readily available. Accordingly, an arrangement has been entered into with a reputable firm of scientific dealers to market pieces constructed exactly according to our specifications. It is our contention that the difficult vacuum operations described can be accomplished more easily by simple combination of these pieces than ordinary distillations can be performed with the usual laboratory equipment.

The manipulation chart of Fig. 19 shows a number of

FIG. 19.



Manipulation chart.

vacuum assemblies grouped round a central oil pump, each set forming an independent unit capable of performing a specific operation. The work of assembly requires nothing more than the sealing of two or three joints.

THE PUMP.*

The rotary, oil sealed, type is preferred, and may be:

(*a*) simply immersed in oil,
(*b*) fitted with separate oil container and provision for separating condensed water.
(*c*) immersed in oil. Oil entering the second (high vacuum) stage is rectified by passage through the first stage.

Our preference is emphatically for type *c* in the laboratory and type *b* for industrial use.

THE PUMP OIL.

Highly rectified phlegmatic hydrocarbon oils are available, and should be used where the highest vacuum is required for physical purposes. Slight contamination with solvent or water vapor reduces these oils to the level of crank case oil. For chemical work, a good summer grade of motor car crank case oil is perfectly satisfactory, and is cheaper. Motor oil frequently changed is far preferable to an expensive filling husbanded beyond its useful life.

THE VAPOR TRAP.

The vapor trap generally consists of a U-tube immersed in refrigerant contained in a dewar vessel. We prefer to utilize the vessel directly by applying wide delivery arms to an expansion at the base. The freezing material is placed in the inner pocket where it remains protected by the operating vacuum; the "cold" can only escape through the gas to be frozen, and thus the trap is economical and of large capacity. The relative merits of liquid air and solid CO_2 as refrigerants are well understood. The former reduces the pressure of water vapor to about .01 mm. in the trap described; liquid

* Reciprocating pumps and even steam ejectors can now give very low pressures. Space is not available to describe the many types.

air reduces it to a very small quantity. Mercury vapor is lowered to one-millionth of a micron by CO_2, and butyl phthalate vapor to an unmeasurable pressure. We recommend solid CO_2 as adequate for chemical purposes.

THE DRYING OPERATION.

Group 1. Powdered minerals, cellulose, wool, gelatin, and many crystalline salts can be dried by vacuum as conveniently as by heating to 110°. The vacuum desiccator *B* connects by a wide tube *C* with the trap *A*, and thence by a narrower tube *D* to the pump. A manometer is generally unnecessary if the joints have been liberally greased with castor oil jelly. Alternatively, the sample is placed in a spring balance and the rate of dehydration plotted against readings from a phthalate manometer.

Group 2. The Industrial Drying Oven.—This assembly can be used for drying or drying and impregnating. Its use will be described in connection with the manufacture of electrical condensers. The oven is a square or cylindrical box fabricated from sheet metal by welding, and preferably tinned inside and out to seal any porous regions. The door is a stout plate machined to bear on to a machined flange 2″ wide attached to the oven. *No bolts* or fasteners should be used, the vacuum itself providing sufficient, very evenly exerted, closing pressure. A flexible rubber gasket moistened with castor oil is desirable.

Attached to the oven by a short, wide tube is a condenser, conveniently made from brass or steel tubing. A cooling coil is inserted and the joints welded and then soldered. The bottom of the vessel is permanently closed except for a drain cock and the top, which may have a glass window, is fastened like the oven. The outside of the joints may be smeared with much castor oil jelly. A comparatively narrow tube is used between the condenser and the pump. The condenser coil may be cooled by ammonia, by chilled brine, or preferably by carbon dioxide. The electrical condensers are stored in a warm place before treatment. They are then suspended in the oven over a vessel of molten wax or chlorinated naphthalene, and the assembly is subjected to vacuum until a pressure below .1 mm. has been reached. The condensers are now

released and allowed to fall into the impregnating material. Air is again admitted to the oven, after which the condensers are withdrawn and drained. The water trapped in the cold condenser is thawed out by steam before the oven is re-used. Notice that the term "oven" is purely complimentary, no heat other than that stored in the materials being required.

Group 3. The Boiling-point Still.—For laboratory distillations down to 2 mm. Hg. The phlegmatic liquid is freed from common volatiles by preliminary exhaustion and is introduced into the distillation flask. The pressure is reduced to the required degree, and the system shut off from the pump. The true distillation pressure is the sum of the pressure at the gauge (*A*) and the automanometer.

Group 4. The Condensation Pump Still.—For pressures down to 1 mm. with external gauge, and from 1 mm. to 0.1 mm. with automanometer alone.

Group 5. Simple Evaporative Distillation.—Most phlegmatic liquids have boiling points so much higher than those of their component materials that simple distillation yields a pure product. The cooling trap is optional and merely precautionary. Pressure reductions to .01 mm. are available and are sufficient for 95 per cent. of this type of work.

Group 6. Evaporative Distillation of Highly Phlegmatic Liquids.—The reduction of pressure is limited only by the nature of the mixture under treatment and the time allotted. A trap of solid carbon dioxide may be applied between the pump and still or the gauge and still. The trap may be of conventional design or may be constructed in an emergency by the simple expedient of packing the refrigerant round the tubes in a paper carton lagged with absorbent cotton, as shown at *A*, A^1.

Group 7. Evaporative Fractionation.—The diagram is self-explanatory.

Conclusion and Acknowledgments.—The combinations of apparatus suggested in the chart are by no means exhaustive. Phthalate condensation pumps may be coupled to vacuum spectrographs and x-ray tubes, and they may be used in the Neon sign industry. Vacuum oven technic may be employed for the cathodic spluttering of metals, the study of shrinkage of fibers and cellulosic sheets on prolonged desiccation, the

regeneration of absorbent materials like silica gel or animal charcoal. The study of "Bakerian Dryness" phenomena in phlegmatic liquids has not yet been attempted. There are many types of research which will benefit when a generalized simple vacuum technic becomes common practice. The methods described in this paper lay little claim to novelty of principle. They have been collected and adapted from the work of many authors and the selection has necessarily been biassed in nature. If the writer has seemed to give undue emphasis to his own favorite modifications of apparatus, it is because his object is to recommend a coherent system of procedure, rather than to cover this wide field impartially.

RESEARCH LABORATORIES,
CHEMICAL DEVICES DEPARTMENT,
November 4, 1931.

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THE SOLUTION OF STEADY-STATE PROBLEMS IN DIELECTRIC, MAGNETICALLY PERMEABLE, AND CONDUCTING MEDIA, WITH SPECIAL REFERENCE TO MATHEMATICAL ANALOGIES BETWEEN MAGNETIC PROBLEMS AND CURRENT-FLOW PROBLEMS.[1]

BY

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Introduction.—It is, perhaps, hardly necessary here to emphasize the importance of the well-known analogue between magnetic problems and problems in the conduction of electricity. The value of the analogue consists in its enabling us to take the results which are worked out in a field where our intuitive instincts have become sharpened by practice, or by a concrete picture of the mechanism of the processes, and import them to another field in which the concepts are apt to appear more abstract on account of the less familiar physical picture we form of the mechanism of the procedure. Thus, a network of wires with electromotive forces in them constitutes not only a problem which has been carried far as regards the details of its many aspects, but one in which our picture of the mechanism of what is happening is very definite and strongly stimulative of our physical intuition. We have found it convenient to introduce such quantities as the resistance of the wire as a whole and to conclude readily how it depends upon its diameter and length. We are not troubled about doubts as to whether the current is uniform over a cross section of a cylindrical wire, even if it is not round. We say, "of course, it is," and if anyone asks us to prove it, we start off with the tremendous confidence that the result must be true. In any case where the current flow departs from the characteristics which it would have in some other customary and simple case, we say, "Well, any fool could

[1] A paper presented before the Division of Engineering and Industrial Research of the National Research Council on November 13, 1931.

see that at once," and we proceed to invent ingenious methods of seeing just why and how it should depart from the simpler conditions. Our intuitions do not immediately suggest to us that the same kind of thing can be done in a magnetic problem where, however, the assignment of the electromotive forces in the circuits is replaced by the assignment of the line integrals of the magnetomotive force around corresponding paths. It is not immediately obvious that quantities in every way analogous to the resistance of the wires as a whole have a significance, and an equally useful one, in the case of bars of magnetizable material in the magnetic problem. The mathematical theory stimulates our intuitions to this view, however, and gives us the confidence necessary to construct "Wheatstone bridges" out of magnetizable bars and magnetomotive forces, in place of copper wires and electromotive forces, and to obtain comparisons of permeabilities in a manner exactly analogous to that adopted for the comparison of resistances. Even the cases where magnetic inductions and field strengths are not proportional have their complete analogue in conductors which do not obey Ohm's law.

The general nature of the results to be presented in this paper is not fundamentally new. Indeed, the mathematical analogy between problems in magnetically permeable media and problems in current flow is one used extensively by engineers. Nevertheless, the treatment of these matters as usually given seems to be rather sketchy and limited, in general, by restrictions such as those implied in Ohm's law, and in the existence of a constant permeability, etc., restrictions which are not fundamental to the spirit of the theory and tend to cloud its exact significance. Then, there are certain matters concerned with the uniqueness of the solutions, and the significance of the conditions which must be assigned to give unique solutions, which can profit by a more careful statement than is usually given. For all such reasons, it has been thought worth while to present the whole theory of the subject in a reasonably consistent form, in the hope that the emphasis on certain features and the development of certain theorems and ideas, some of which are new, may stimulate a more rigorous understanding of the significance of the field concerned.

We shall first make a comparison of the solutions of

A. Problems of current flow in conducting media;
B. Problems of electrostatic fields in dielectric non-conducting media;
C. Problems of magnetostatic fields in magnetically permeable media.

In all these cases, we shall leave as much generality as possible as regards variation of the constants of the media from place to place. Finally, we shall discuss the problem in which the medium possesses conductivity, dielectric permeability and magnetic permeability.[2]

Definitions of Symbols.—The following scheme of notation will be used: $\boldsymbol{E}$, for electric-field strength; $\boldsymbol{D}$, for dielectric displacement; $\boldsymbol{H}$, for magnetic-field strength; $\boldsymbol{B}$, for magnetic induction; $\boldsymbol{u}$, for current density; ρ, for volume-charge density, and σ, for surface-charge density. Other quantities used will be defined as they occur. Although the dielectric polarization (electrostatic moment per unit volume), $\boldsymbol{P}$, and the magnetic intensity (magnetic moment per unit volume), $\boldsymbol{I}$, will not play any fundamental rôle in our discussion, it may be well to recognize that, in Heavisidean units,

$$\boldsymbol{D} = \boldsymbol{E} + \boldsymbol{P}; \qquad \boldsymbol{B} = \boldsymbol{H} + \boldsymbol{I}.$$

In ordinary electrostatic and magnetostatic units, 4π precedes the $\boldsymbol{P}$ and the $\boldsymbol{I}$.

The Mathematical Equations.—It will be convenient so far as possible to carry side by side, in three columns, the equations corresponding to the cases *A*, *B*, *C*, cited above. As fundamentally governing the course of events in the media, we have:

A	*B*	*C*
$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}=\alpha$ (1)	$\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}+\frac{\partial D_z}{\partial z}=\rho$ (2)	$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=\beta$ (3)

Here (1) represents the equation of continuity. The right-hand side is zero in practical cases. However, the mathe-

[2] In order to make this paper readily readable to as large a group of engineers as possible, the mathematical procedure will be presented in as elementary a manner as is possible.

matical theory associated with the equations we are using [3] would permit the spontaneous generation of electricity in the element of volume, so that we shall retain α in principle, with the understanding that, if present in a problem, its magnitude must be assigned at each point in the media.

Equation (2) is simply Gauss' theorem, with ρ as the volume density of charge.

In (3) div $\boldsymbol{B}$, and consequently β are always zero if we consider the magnetic phenomena as arising entirely from electron currents. Thus, if a so-called permanent magnet is embedded in a magnetically permeable medium, and if $\boldsymbol{B}$ refers to the total magnetic induction, including that of the permanent magnet, then div $\boldsymbol{B}$ is always zero. It may be a matter of mathematical convenience, however, to regard the permanent magnet as a separate magnetizing entity, and express its action in terms of its permanent moment per unit volume $\boldsymbol{I}_0$. As a result of a well-known mathematical theorem, we then know that the distribution $\boldsymbol{I}_0$ is the equivalent of a volume distribution of *magnetism* M per unit volume, and a surface distribution m, per unit of area, where

$$M = -\,div\,\boldsymbol{I}_0; \qquad m = \boldsymbol{I}_n,$$

and $\boldsymbol{I}_n$ is the value of the normal component of $\boldsymbol{I}$ at the boundary of the medium. We may then regard these magnetic "charges" as operative in a way analogous to, and according to the same laws as, electric charges in a dielectric medium. They operate to magnetize the medium in which the permanent magnet is embedded and, incidentally, the material of that magnet itself. In this case, β corresponds to the volume density M of the "fictitious" volume distribution of the permanent magnetism. Moreover, $\boldsymbol{B}$ refers to that portion of the complete magnetic induction which omits the part $\boldsymbol{I}_0$ due to the permanent magnetization. A fuller clarification of this matter is relegated to an appendix.

In addition to the equations (1), (2), (3), we have functional relationships between $\boldsymbol{u}$ and $\boldsymbol{E}$, $\boldsymbol{D}$ and $\boldsymbol{E}$, $\boldsymbol{B}$ and $\boldsymbol{H}$, which we may designate symbolically as

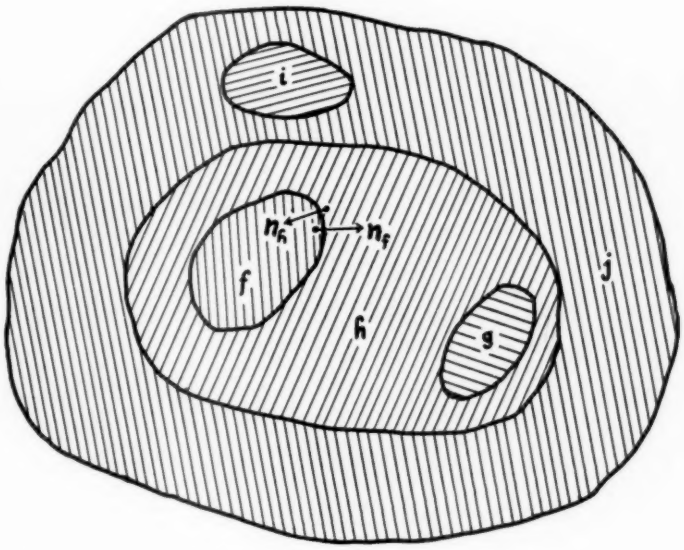
$$\boldsymbol{u} = f_1(\boldsymbol{E}) \quad (4); \quad \boldsymbol{D} = f_2(\boldsymbol{E}) \quad (5); \quad \boldsymbol{B} = f_3(\boldsymbol{H}) \quad (6)$$

[3] The electromagnetic equations would deny the existence of α, however.

In general, the function f may change as we cross boundaries separating the different kinds of media from each other. In the case of Ohm's law, $f_1(\boldsymbol{E})$ becomes $k\boldsymbol{E}$ where k is a constant. In the case of a dielectric constant K independent of $\boldsymbol{E}$, $f_2(\boldsymbol{E})$ becomes $K\boldsymbol{E}$; and in the case of a permeability μ independent of $\boldsymbol{H}$, $f_3(\boldsymbol{H})$ becomes $\mu\boldsymbol{H}$.

For generality, we shall consider a case where media of various kinds are embedded in one another. The properties of any one medium may vary from point to point, and there

FIG. 1.



may be discontinuities in the quantities concerned at the boundaries of the media. Considering any one of the vectors, say $\boldsymbol{u}$, we shall mean by $\boldsymbol{u}_{nf}$ the component of $\boldsymbol{u}$ at a point on the boundary of f, resolved along the direction of the outwardly drawn normal from f. By $\boldsymbol{u}_{nh}$, we shall mean the component of $\boldsymbol{u}$ at a point on the boundary of h resolved along the direction of the outwardly drawn normal from h. At a point on the geometrical boundary between the two media, these two normals are in opposite directions, as indicated by n_f and n_h, in Fig. 1. The geometrical boundary

represents two surfaces regarded as boundaries of the media f and h. One is the "outside of the inside," and the other is the "inside of the outside." u_{Tf} and u_{Th} shall represent the tangential components at the boundaries in the two media.

In general, the outer bounding surface of all the media will be at infinity, but it need not be. The fact that there may be media outside such a boundary as the outer boundary of j, Fig. 1, need not prevent us from limiting our interest to a discussion of the phenomena inside that boundary. The solution of our problem will, however, then require the specification of finite values to the magnitudes of certain quantities on the boundary.

Now for the solution of our problem, we shall assume (1), (2), or (3) as the case may be, and (4), (5), or (6), and we shall assign at each point in space, for the three types of problem A, B, C, respectively the magnitudes of:

A		B		C	
$\int_{\odot} E_s ds$ [4]	(7)	$\int_{\odot} E_s ds$	(8)	$\int_{\odot} H_s ds$	(9)

At each point of the boundary between two media denoted by subscripts p and q, we shall assign the values of

$u_{np} + u_{nq}$ $E_{Tp} - E_{Tq}$	(10)	$D_{np} + D_{nq}$ $E_{Tp} - E_{Tq}$	(11)	$B_{np} + B_{nq}$ $H_{Tp} - H_{Tq}$	(12)

At each point of the external boundary, we shall assign the values of

u_n	(13)	D_n	(14)	B_n	(15)

We shall prove below (uniqueness theorem No. 1), that if we have a solution of (1), (2), or (3), as the case may be, consistent with (4), (5), or (6), as the case may be, and giving the corresponding assigned magnitudes, as indicated above, then the solution is unique.

[4] This implies the assignment of the line integral of the vector around all closed curves in the region. The assignment of the line integral in this manner is sufficient to determine the curl of the vector at each point in the region, but the converse is not necessarily the case. Thus, in the region *outside* of a wire carrying a current, the magnetic field has zero curl at all points, but it has a finite line integral for a closed curve encircling the wire.

There is an alternative to the assignments (10), (11), (12), and a corresponding alternative to the assignments (13), (14), (15). These alternatives represent, in fact, the customary assignments when the line integrals of $\boldsymbol{E}$ or $\boldsymbol{H}$, as the case may be, are zero, so that the vectors are completely derivable from a potential. The situation is as follows:

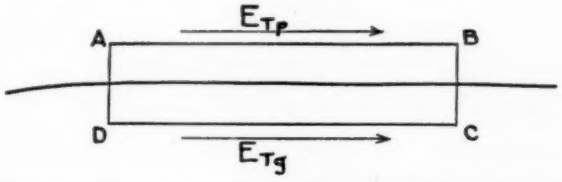
Suppose that from the total field in each region we have split off and specified any portion which provides for the values of all the line integrals around closed paths.[5] Then we know that the remainder of the field is determinable entirely from a potential φ. We shall show (uniqueness theorem No. 2) that by splitting off and specifying a portion of the field in the above manner, and then specifying the values of φ at each point on each side of each boundary, and at each point of the external boundary we can dispense with the assignment of (10) and (13), or (11) and (14), or (12) and (15), as the case may be.

Remarks on the Significance of the Assigned Surface Conditions.—It is customary to give the conditions (10)–(12) in the form:

$$\left.\begin{aligned} u_{np}+u_{nq}&=0\\ E_{Tp}-E_{Tq}&=0\end{aligned}\right\}\ (16)\qquad \left.\begin{aligned} D_{np}+D_{nq}&=0\\ E_{Tp}-E_{Tq}&=0\end{aligned}\right\}\ (17)\qquad \left.\begin{aligned} B_{np}+B_{nq}&=0\\ H_{Tp}-H_{Tq}&=0\end{aligned}\right\}\ (18),$$

and it is customary to attempt to prove them. The greatest offense in this matter is committed in relation to the attempt to prove $E_{Tp} - E_{Tq} = 0$ and $H_{Tp} - H_{Tq} = 0$. It is customary to draw a rectangle A, B, C, D, encircling a portion of the boundary, Fig. 2. Then the argument is as follows:

FIG. 2.



[5] Thus, in a problem where the electric field $\boldsymbol{E}$ was determined in part by batteries connecting conductors, and in part by a rate of change of magnetic field, we could calculate the latter part as the part which would be produced if the changing magnetic field were the only influence operating. In other words, starting with all the conductors, etc., present, but with no field anywhere, we could calculate the part of the field $\boldsymbol{E}$ produced by superimposing the changing magnetic field alone.

Work done in carrying unit charge around rectangle $= \overline{AB}E_{Tp} - \overline{DC}E_{Tq}$, the work done in going from A to D being canceled by the work done in going from C to B either by both being zero, on account of the shortness of the paths as compared with $\overline{AB}$, or by their being equal in the event of a finite amount of work being done in crossing the boundary. The argument is then to the effect that since the total work in going around the rectangle is zero, we must have $E_{Tp} - E_{Tq} = 0$. Now it is quite unjustifiable to assume that the work done in crossing the boundary at A is equal to that done in crossing at C, or indeed that the work done in going around the whole rectangle is zero. Moreover, the objection to the assumption is not a mere quibble, for one can realize an actual physical case where the assumption is not true. Thus, consider the Peltier effect across the boundary of two media, and suppose that the temperature varies over the boundary between the two media. Then, it will be obvious that in such a rectangle as is drawn in Fig. 2, the Peltier drop from A to D will not be equal to that from B to C. Moreover, the work done in going right around the circuit will not be zero. Furthermore, the field E_{Tp} will not be equal to E_{Tq}, but will, as a matter of fact, be opposite in direction. Analogous remarks apply to an attempt to prove that $H_{Tp} - H_{Tq} = 0$. The result will actually be false if there is a finite surface sheet of current on the surface. In such a case, there will be line integral of H taken around such a rectangle as A, B, C, D, Fig. 2.[6] However, the assumptions of zero values to $H_{Tp} - H_{Tq}$, or $E_{Tp} - E_{Tq}$ are not necessary for the solutions of problems. All that is necessary is that the magnitudes of these quantities shall be assigned. If they are assigned as zero, the assumption is the analytical equivalent of the restriction of our interest to those cases where such phenomena as surface current sheets, or thermoelectric phenomena across the boundary are absent.

The assignment $u_{np} + u_{nq}$ as zero instead of a finite quantity is merely the limitation of our interest to the practical case of absence of sources and sinks. The assignment of a

[6] I find that in H. Pender's book, "Electricity and Magnetism for Engineers," Vol. 1, p. 209, the possibility of H_{Tp} being unequal to H_{Tq} as a result of a current sheet is admitted.

finite value to $D_{np} + D_{nq}$ merely recognizes the possibility of a finite surface density of charge. The assignment of a finite value to $B_{np} + B_{nq}$ has a significance in the case, as described above, where we find it convenient to express the effects of permanent magnets in terms of their fictitious volume and surface charges. Then $B_{np} + B_{nq}$ represents minus the surface density of fictitious "magnetic charge," the minus occurring on account of the way in which we have drawn the normals in Fig. 1.

Uniqueness Theorem No. 1.—We shall take as an example the case of current flow. For the other cases, the theorem is exactly analogous. We then proceed to prove that if we have a solution of (1) consistent with an assigned distribution of values of α, and if the solution satisfies (10) and (13), subject to (4), then it is unique.

Suppose there are two solutions for $\boldsymbol{u}$, and $\boldsymbol{E}$. Let the difference of the two be given by $\boldsymbol{w}$ and $\boldsymbol{e}$. Then, since both solutions satisfy the assigned conditions,

$$\left.\begin{array}{lll} \text{At every point in the regions} & \left\{\begin{array}{ll} div\ \boldsymbol{w} & = 0, \\ \int_{\odot} eds & = 0, \end{array}\right. & \\ \text{On the boundaries} & \left\{\begin{array}{ll} w_{np} + w_{nq} & = 0, \\ e_{Tp} - e_{Tq} & = 0, \end{array}\right. & \\ & w_{nb} = 0, & \end{array}\right\} \tag{19}$$

where w_{nb} indicates the value of w_n at a point on the external boundary. Since $\int eds = 0$ for all closed paths, $\boldsymbol{e}$ must be derivable from a potential ψ. Consider Green's lemma as applied to three components of a vector W:

$$\int\int\int \text{div } \boldsymbol{W}\, d\tau = \int\int W_n dS.$$

Take

$$W_x \equiv \psi w_x; \qquad W_y \equiv \psi w_y; \qquad W_z \equiv \psi w_z.$$

Then

$$\int\int\int \psi \text{ div } \boldsymbol{w} d\tau + \int\int\int (\boldsymbol{w} \text{ grad } \psi) d\tau = \int\int \psi w_n dS.$$

The surface integral is, of course, taken in the complete sense in which each geometrical surface except the external one is associated with two boundaries as formerly indicated. For this reason, we shall write the surface integral in another form in which each element of the geometrical surface contributes only once, but associated with that element the values of ψw_n will be included for each side of the surface as implied by subscripts p and q. Then, remembering that div $\boldsymbol{w}$ is zero, we have

$$\int\int\int (\boldsymbol{w} \text{ grad } \psi) d\tau = \Sigma \int\int (\psi_p w_{np} + \psi_q w_{nq}) dS + \int\int \psi_b w_{nb} dS, \quad (20)$$

the last integral corresponding to the external boundary as indicated by the subscript b in w_{nb}. Since $w_{nb} = 0$, we have

$$\begin{aligned}\int\int\int (\boldsymbol{w} \text{ grad } \psi) d\tau &= \Sigma \int\int (\psi_p w_{np} + \psi_q w_{nq}) dS \\ &= \Sigma \int\int \psi_p (w_{np} + w_{nq}) dS - \Sigma \int\int w_{nq} (\psi_p - \psi_q) dS. \quad (21)\end{aligned}$$

Now $w_{np} + w_{nq} = 0$. Hence, the first term on the right-hand side of (21) is zero. Moreover, since $e_{Tp} - e_{Tq} = 0$, at each point of a boundary, $\psi_p - \psi_q$ must be constant over any single geometrical surface. Thus,

$$\int\int w_{nq} (\psi_p - \psi_q) dS = (\psi_p - \psi_q) \int\int w_{nq} dS. \quad (22)$$

Now consider any one of the regions, such as f, Fig. 1, bounded completely by a surface. Then, if subscript f refers to this region, Green's lemma gives

$$\int\int w_{nf} dS = \int\int\int \text{div } \boldsymbol{w}_f d\tau.$$

But div $\boldsymbol{w}_f$ is zero everywhere. Hence

$$\int\int w_{nf} dS = 0.$$

Now consider such a region as h. Since at each boundary $w_{np} + w_{nq}$ is zero, the fact that the surface integral over the boundaries of regions of the class f are zero insures that the integrals over the internal boundaries of the regions of the class h shall be zero. But, using Green's lemma again, the fact that div $\boldsymbol{w}$ is zero in h insures that the sum of the surface integrals of w_n over the internal and external boundaries of h shall be zero. We have found that the surface integral is zero over the internal boundary. Hence, it is also zero over the external boundary. The continuity of w_n across the boundary of h then insures that the surface integral of w_n for the outside of the boundary of h shall be zero, and so on.

We thus see that all the surface integrals of the type occurring on the right-hand side of (22) are zero. Hence the integrals of the type occurring on the left-hand side of (22) are zero. Hence the second summation in (21) is zero, and since we have proved that the first summation is zero we have

$$\int\int\int (w \text{ grad } \psi) d\tau = 0,$$

i.e.

$$\int\int\int \left(w_x \frac{\partial \psi}{\partial x} + w_y \frac{\partial \psi}{\partial y} + w_z \frac{\partial \psi}{\partial z} \right) d\tau = 0. \qquad (23)$$

Now, in spite of the fact that $\boldsymbol{w}$ and grad ψ represent the difference of the corresponding vectors for two *different* solutions, tentatively supposed possible, the form of the functional relationships (4) does not change from one solution to the other. This relationship is a characteristic alone of the particular medium to which it refers. In fact, (4) provides a definite functional relationship between any change of $\boldsymbol{E}$ and the corresponding change of $\boldsymbol{u}$. This change of field concerned in our case is represented by grad ψ, and the change of $\boldsymbol{u}$ by $\boldsymbol{w}$. If then it is always true that increase of $\boldsymbol{E}$ increases $\boldsymbol{w}$, or even if it is *always* true that increase of $\boldsymbol{E}$ decreases $\boldsymbol{w}$, then

$$w_x \frac{\partial \psi}{\partial x}, \quad w_y \frac{\partial \psi}{\partial y}, \quad \text{and} \quad w_z \frac{\partial \psi}{\partial z}$$

have the same sign as one another and, moreover, have the same sign at all points in all the regions. We shall exclude

the purely artificial case where increase of $\boldsymbol{E}$ sometimes increases $\boldsymbol{w}$ and sometimes diminishes it. Common sense would also lead us to exclude likewise the case where increasing $\boldsymbol{E}$ diminished $\boldsymbol{w}$. However, *analysis* does not require even this restriction to conclude, as we have, that each of the quantities $w_x \frac{\partial \psi}{\partial x}$, $w_y \frac{\partial \psi}{\partial y}$, and $w_z \frac{\partial \psi}{\partial z}$ must be of the same sign at all points, or, of course, zero. But in view of this, the only way in which (23) can hold is by each of the quantities being zero at each point in the space. Hence,

$$\left.\begin{aligned} w_x = 0 \quad &\text{or} \quad \frac{\partial \psi}{\partial x} = 0, \\ w_y = 0 \quad &\text{or} \quad \frac{\partial \psi}{\partial y} = 0, \\ w_z = 0 \quad &\text{or} \quad \frac{\partial \psi}{\partial z} = 0. \end{aligned}\right\} \tag{24}$$

However, apart from trivialities, the functional relationship (4) provides for no change of $\boldsymbol{w}$ without change of $\boldsymbol{E}$ and vice versa. Both changes are zero if either of them is zero. Hence, $\boldsymbol{w}$, and grad ψ are zero everywhere, and the uniqueness of the solution is proved. In exactly the same manner, we establish the corresponding theorems for the cases found upon equations (2) and (3), under the corresponding conditions as cited above.

Uniqueness Theorem No. 2.—We proceed to prove that if we have a solution consistent with an assigned distribution of values of α, and if a portion of the field which provides for all of the line integrals has been separated off, so as to leave a portion giving no line integral and, therefore, derivable from a potential φ, and if the solution fits an assigned distribution of the potential over the boundaries, then the solution is unique.

Again, let there be two solutions, if possible, and let them differ by $\boldsymbol{w}$ and $\boldsymbol{e}$. The latter quantity must, of course, be of the form grad ψ. We proceed exactly as in the last theorem, until we arrive at (20). Now, however, the conclusion is simpler than before. For, since the potential is assigned on both sides of all the surfaces and at the external

boundary, ψ is zero at all points on the boundaries. Hence, we arrive at (23) immediately, and so at (24), and at the conclusion that $\boldsymbol{w}$ is zero and grad $\psi = 0$. We conclude that ψ must be at most a constant. But since it is zero on the boundaries, the constant is zero. Hence, the uniqueness is proved.

Case Where the Medium Possesses Conductivity, Dielectric Properties, and Magnetic Properties.—In this case, we solve (1), subject to the assignments (7), (10), and (13), and subject to (4). Or, as already shown, we may replace (10) by the assignment of the appropriate potentials at the boundaries.

The solution becomes uniquely determined quite apart from the existence of any special dielectric or magnetic properties. Through the relation (4), we can now determine $\boldsymbol{E}$ at all points, and, through (5), we can then obtain $\boldsymbol{D}$. Equation (2) then enables ρ to be determined. We do not now have to *assign* (11) or (14) at the boundaries, because the values of these quantities become automatically determined. The quantity $D_{np} + D_{nq}$ determines the surface charge densities on the boundaries between two regions. It will be observed that in this case we do not have to *solve* (2). The conductivity of the medium permits the movement of electricity in such a way as to *create* a distribution ρ which permits (2) to be satisfied, and analogous remarks pertain also to the surface charges.

We may now superimpose upon the solutions obtained the complete problem as specified by (3), with its accompanying assignments to provide for the magnetic aspect of the problem. The solution of the conduction problem does not interfere with the magnetic solution in any way, except for the magnetic fields generated by the currents and except in so far as we may have invoked changing magnetic fields in the production of the electrical fields. The inclusion of the magnetic fields due to the steady currents presents no difficulty. They are definitely calculable and are simply to be included in the magnetic problem. The invocation of magnetic fields changing with time would be, in principle, out of the realm of the problems we have set ourselves, i.e., the problems for steady states. In cases where the time rates of change of the quantities concerned are such that the current distribu-

tion, etc., adjust themselves, approximately, to the steady-state conditions at each instant, we may permit such time rates of change of the magnetic fields and solve the electrical problem independently of the magnetic problem, to the extent that we may neglect the effect, in the magnetic problem, of the magnetic fields generated by the currents and changing electric fields in the electrical problem. The inclusion of these effects is, of course, possible, but it takes the problem out of the category which it is the prime function of this paper to consider, viz., problems in which the magnetic and electrical portions are sufficiently separable to enable us to stress the analogy between the two.

Note on the Mathematical Significance of a Battery.—From a mathematical standpoint, a battery may be regarded as a system in which there is a discontinuity in the potential at the boundary of each metal and the liquid, and another discontinuity between the junction of the two metals. The field is strictly derivable from a potential, and the line integral of the field around the whole circuit is, strictly speaking, zero. However, the currents in the media are the same as they would be if we had a field which, at all points in these media, was the same as the actual field, but which was not associated with finite discontinuities in potential at the boundary. Such a field would have a finite line integral taken around the circuit. Thus, in introducing a battery, we may either represent it as a device in which we have the discontinuities of potential above referred to, or we may treat it as a device for producing a line integral of electric field which, as regards production of the currents, is the equivalent of the fields obtainable from the potentials. The line integral in question is, of course, the electromotive force.

APPENDIX.

Note on the Possibilities as Regards the Representation of the Effects of Permanent Magnetism.—It is a well-known fact that any distribution of polarization $\boldsymbol{I}_0$ contributes to the field $\boldsymbol{H}$ at a point outside it, a portion which is calculable from a fictitious volume density M of magnetism and a fictitious surface density m of magnetism spread over the boundary of the region, provided that

$$M = -\operatorname{div} \boldsymbol{I}_0$$

and

$$m = I_n.$$

Moreover, the field, when so calculated for a point inside the medium, gives the field as ordinarily defined in the medium, and as corresponding to distribution I_0.

Now, if for the moment we take $\bar{B}$ as representing the total magnetic induction and $\bar{I}$ as representing the total polarization, we have always:

$$\begin{aligned} \text{div } \bar{B} &= 0, \\ \bar{B} &= H + \bar{I}, \end{aligned}$$

where H represents the total field.

Now let I_0 be a part of $\bar{I}$ which is separated off as representative of permanent magnetic moment, so that

$$\bar{B} = H + I + I_0,$$

where I is defined as $\bar{I} - I_0$.

If, then, we define B as

$$B \equiv H + I,$$

we have

$$0 = \text{div } \bar{B} = \text{div } B + \text{div } I_0.$$

Hence,

$$\text{div } B = M. \tag{25}$$

Also, for the boundary between two media denoted by subscripts p and q, we have for the normal component:

$$\begin{aligned} \bar{B}_{np} + \bar{B}_{nq} &= 0, \\ (B_{np} + I_{0np}) + (B_{nq} + I_{0nq}) &= 0, \\ B_{np} + B_{nq} &= -(I_{0np} + I_{0nq}) \\ &= -(m_p + m_q). \end{aligned} \tag{26}$$

The minus sign results from the direction which we have taken for the normals indicated in Fig. 1. In the case where only one of the adjoining media possesses permanent polarization, only one of the m's occurs. Equation (25), of course, interprets the significance of β in (3), and (26) interprets the significance of the assignment of $B_{np} + B_{nq}$ in (12).

Finally, it may be remarked that even the material of the permanent magnet may possess an induced $\boldsymbol{B}$ and an induced $\boldsymbol{I}$, and the analysis is in a position to recognize this automatically. Our permanent $\boldsymbol{I}_0$ having been assigned, and replaced by its fictitious charges, the medium of the permanent magnet becomes subjected to their influence. An ordinary permanent magnet possesses both $\boldsymbol{I}$ and $\boldsymbol{I}_0$.

BARTOL RESEARCH FOUNDATION
OF THE FRANKLIN INSTITUTE,
November 18, 1931.

THE ELECTRIFICATION OF THE PENNSYLVANIA RAILROAD.*

BY

J. V. B. DUER,

Electrical Engineer, The Pennsylvania Railroad.

I am glad to have had an opportunity to talk to you tonight on the subject of The Pennsylvania Railroad electrification. This is, to me, an always interesting subject, and as a great deal has been said and written about it, I hope you will excuse me if I repeat some of the things with which you are already familiar, because I believe that a general description of the project and the steps which led up to it may prove of more interest to you than a detailed discussion of some particular technical feature which, while it might prove of great interest to some of you, would possibly not be of very much interest to the rest.

As you know, The Pennsylvania Railroad has moved trains by electricity for many years, and I will not attempt to deal with this phase of the subject except to say that our first electrifications used the standard direct current system in use at that time, which, after much experience and investigation, has been gradually replaced by the 11,000-volt, single-phase system, which is now our standard. The first installation on our railroad of the 11,000-volt system was made when we electrified to Paoli in 1915, and it has been gradually extended until, at the present time, we operate trains by this system between Trenton and Wilmington, between Philadelphia and Paoli, Philadelphia and Norristown, Philadelphia and Chestnut Hill and West Chester.

In 1928 President Atterbury announced that a program of electrification between New York and Washington had been authorized, and that this program contemplated electrification of the through freight and passenger service between these cities. It is upon this program that we are now working, and

* Presented at a meeting held Thursday evening, October 29, 1931.

to give you a picture of what has already been accomplished, I would say that the portion of the electrification between Sunnyside yard and Manhattan transfer, which replaces our present direct current operation of Pennsylvania Railroad trains in the Pennsylvania terminal area, is about 95 per cent. completed, and will be placed in operation with alternating current locomotives some time after the first of the year.

Between Manhattan Transfer and New Brunswick the electrification is approximately 50 per cent. completed, and we hope to place this in operation during the coming summer. Between New Brunswick and Trenton the work is actively under way on the foundations for the catenary structures and the preparation of the substation sites, and we hope to have this length complete for operation early next summer. As you know, the link between Trenton and Philadelphia was completed in 1930, and we expect, therefore, to run through trains from New York to Philadelphia during the coming summer.

Between Philadelphia and Wilmington, the line has already been electrified, and between Wilmington and Washington work is actively under way on the installation of the foundations for the catenary structures and the construction of the duct line in which we shall place our communication and signal circuits to prepare for the electrification work.

The link between Wilmington and Washington we expect to complete in 1933, so that the electrification of our through freight and passenger service between New York and Washington should be in full operation during that year.

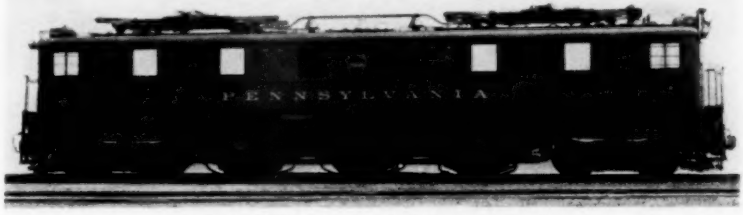
You will readily appreciate that with a project of this magnitude these dates must be, for the present, at least, tentative; but, at the present time, there seems to be no reason why we should not meet them.

In addition to the electrification of the railroad and the installation of the substations necessary to supply it with power, as well as the necessary transmission lines to distribute this power between the substations along the railroad, power contracts have been entered into for the supply of current at Philadelphia, Baltimore, and Perryville, and power under these contracts will be available by the time the electrification work requires it.

It has also been necessary to design and build for this work 105 passenger locomotives, 60 freight locomotives, 65 switching locomotives, and 114 multiple unit passenger cars. The locomotives are under construction at the present time and will be available for service by the time they are needed in the electrification program, and work will be begun on the equipping of some of our passenger cars for suburban service so that they will be available at the time they are needed.

It might interest you to know that before this program was started we had 1,143 miles of electrification on The Pennsylvania Railroad, including that on the Long Island Railroad; 71 substations, with an installed kilowatt capacity of 539,500, and were using to operate this electrification 465,000,000 kilo-

FIG. 1.



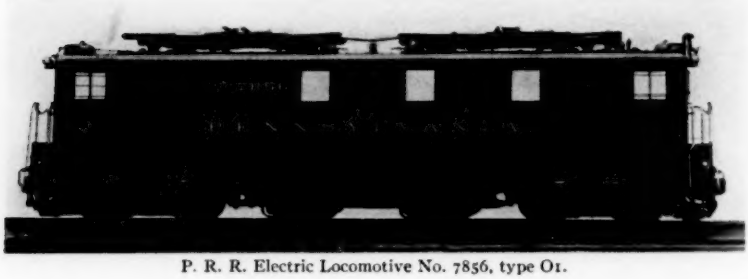
P. R. R. Electric Locomotive No. 7899, type P5.

watt hours per year, using 84 electric locomotives and 1,523 multiple unit cars. After the electrification is completed between New York and Washington in 1933, we shall have 2,210 miles of electrified track; 99 substations, with an installed capacity of 1,374,500 kilowatts; 1,681 miles of single circuit transmission line, and 1,686 miles of catenary circuit, over which electrification we will use 136 passenger electric locomotives, 86 freight electric locomotives, 98 switching electric locomotives, and 1,637 multiple unit cars, and will require 1,185,000,000 kilowatt hours per year for this operation.

While figures often mean very little in the description of a project, I think that these figures will give you a concrete picture of our present and future electrification in a way which could hardly be secured otherwise.

Some of the features of this electrification are of outstanding interest and, while briefly stated, represent the result of many years experience and study. As an example, our electric locomotives are of three different classes: passenger, freight, and shifting. The passenger and freight locomotives, although of widely varying speed and tractive effort characteristics, use practically identical electrical apparatus, and in the two types of passenger locomotive, practically identical mechanical apparatus. The freight locomotives, while different in mechanical design to adapt them better to the different requirements of their service, use electrical equipment practically identical with that on the passenger locomotives. Furthermore, all of this electrical equipment, although manufac-

FIG. 2.



P. R. R. Electric Locomotive No. 7856, type O1.

tured by two different electric companies, is interchangeable, part for part, so that any piece of electrical apparatus of any locomotive may be taken out and replaced by a similar piece of apparatus of the other company's make.

It might also be of interest to state that, whereas it has been, heretofore, necessary for us to use a jack shaft and side rods in order to secure motor capacity to handle the high tractive efforts available with the wheel loads which are in common use on our railroad, it is now possible by developments in the single phase motor to eliminate the jack shafts and side rods, and provide between the driving wheels of the locomotive sufficient motor capacity to handle the tractive effort made available by the weight on drivers. It might also be of interest to know that the electric locomotives throughout

are mounted on anti-friction bearings. This includes the journals of the locomotive and the bearings of the motors, with the exception of the quill bearings in passenger locomotives and axle bearings on freight locomotives, which are of the sleeve type.

A composite design of electric equipment has also been developed for the switching locomotives, and while it could not be made identical with that of the passenger and freight locomotive equipments, by reason of the great difference of service requirements, is interchangeable to a great extent with other locomotive equipment and with car equipment.

FIG. 3.

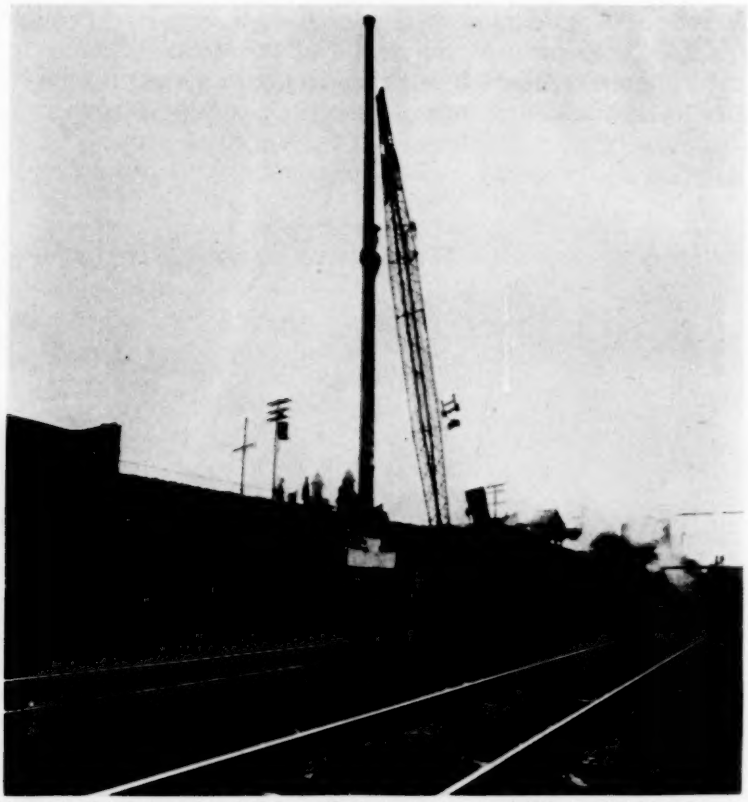
P. R. R. Multiple Unit Car, type MP54-E2.

Our new multiple unit cars will also be fitted with a composite design of equipment, which while being designed to run in trains with our existing multiple unit equipment, will have all the modern characteristics of design made available by work which has been done on the electrical equipment of the electric locomotives.

Similar research work and experiment has resulted in great development in our transmission line and substation designs. Our transmission voltage has been increased from 44,000 in the initial installations to 132,000 in our latest installation.

The substations have been changed from those of an indoor type to outdoor type, permitting considerable increase in capacity at slight increase in cost, and reduction in maintenance expenses. Our substations have been laid out with a mini-

FIG. 4.



Old type derrick setting poles—in Trenton Yard, showing how derrick blocks adjoining track when setting poles.

mum of high tension switching equipment, no oil switches being used on these circuits except at points where it is necessary automatically to separate the railroad into sections, all the supply and protection of the circuits being handled on the low tension side of the transformers in the substations, and

on the low tension side of the step-up transformers at the supply points.

The transformer circuit breakers, together with the bus tie breakers, provide complete protection of the transformers against internal trouble and bus short circuits, as well as against continuous overload due to unexpected peaks, high resistance grounds, or other troubles of this character.

FIG. 5.

Special derrick used for erecting electrification poles, showing how design of derrick prevents blocking of adjacent track.

The trolley circuit breakers feeding the trolleys from a bus are of the high speed type, giving complete operation under short circuit and overload conditions, including selectivity of the circuit, within one cycle.

The overhead construction, including trolley wires, auxiliary wires, catenaries, cross catenaries, and transmission lines, are of non-corrosive material, such as bronze or copper, thus reducing to a minimum the amount of maintenance necessary and making the railroad fully available for service. The clips which hold the hangers to the auxiliary and to the messenger

wire and those connecting the auxiliary to the trolley wire are riveted together instead of being bolted, as heretofore, thus providing a construction which remains tight under vibratory conditions and tends toward a minimum of maintenance.

The bonding of the track rails has been thoroughly reviewed and a pin type of bond developed. This bond is similar in all respects to that used in steam territory for signal circuits, and can be applied and removed by the use of a hammer without other tools. Its use has very materially reduced maintenance in electrified territory by keeping the bonding forces to a minimum.

There are other points about the electrification program which may be of interest and which involve the application of electrification to the railroad. Most of these have been discussed in previous articles, but it may interest you to give you a brief resume of what they are. Two things are of prime importance in any electrification program: One is to decide what work on the railroad is to be done at the same time as the electrification or prior to it; and the other is to be sure that the local conditions are such as to permit the electrification to be made and the other changes to be taken care of.

In other words, it is necessary to study carefully and foresee changes which should be made at the time of the electrification, and it is also necessary to review carefully the real estate situation along the railroad with a view to acquiring any additional property needed in plenty of time.

It is also very desirable to review carefully the question of special equipment needed for electrification purposes and provide it in ample time, such as special types of derricks, concrete trains, tractor derricks for erecting substation steel, and various other equipment which would not ordinarily be carried by the contractor who would handle work of this character, and which would have to be provided by the railroad. Next in importance is a careful study of the railroad to determine what, if any, overhead bridges will have to be raised to provide for trolley wire clearance, what tunnels will have to be revised to accomplish the same result, and what tracks in yards and sidings will have to be included in the electrification program in order to accomplish the desired results.

After these questions have been taken care of, a suitable working program must be made to provide for the ordering of all material in an orderly manner, and for the delivery of this material at such times as will permit it to assume its proper place in the construction schedule. Questions of design of apparatus and equipment should be gotten out of the way early in the proceedings, so that no questions of this character will tend to interfere with the orderly procedure of the work once it is started.

Last, but not least, a construction program should be laid out so that the work will approach completion in all branches at as nearly as possible the same time, thus avoiding to the greatest extent possible the investment of capital in idle equipment and material.

New Wind Vane on Airways. (U. S. Department of Agriculture, Clip Sheet No. 706.) Watchers of the weather along the airways of the United States may sit comfortably indoors with no danger of missing the slightest shift in the direction of the wind, accurate information on which is vitally important to aviators. The instrument which has made this possible—a wind vane bearing devised some months ago by B. C. Kadel, chief of the instrument division, Weather Bureau—has proved its worth and is being sent out in increasing numbers to the weather-observation stations along the main lines of air transportation.

The device consists of an aluminum wind vane mounted on the roof and connected by wires with a series of four electric lamps in the room below. When the vane swings to the north an electric current passes down one of the wires to light the lamp that spells "north" to the observer. When it points to the south that circuit is closed and another wire and lamp come into the picture; and so on for all four points of the compass. Often, of course, two lamps are lighted at the same time, indicating that a northeaster, or some other 2-direction wind, is blowing.

The outstanding feature of the new wind vane, is the extreme simplicity of its design, which makes possible its construction at about one-fourth the cost of those installed at the larger Weather Bureau stations, where more elaborate wind measurements are taken. Observations made with this vane, supplemented by velocity and other data, are transmitted by wire and radio at stated intervals over a wide network to guide pilots in taking off and landing.

R.

THE EFFECT OF ELECTRON ATTACHMENT ON THE ION MOBILITY CURVES IN THE ZELENY AIR BLAST METHOD OF ION MOBILITY MEASUREMENT.

BY

LEONARD B. LOEB AND NORRIS E. BRADBURY.

ABSTRACT.

In a recent paper Professor John Zeleny published curves obtained for ion mobilities at different ion ages in relatively pure N_2 using his classical air blast method. Two features are of especial interest; one the apparent gradual displacement of the negative ion peak to lower mobilities as the ion age increases and the other a marked asymmetry of the electron peak on the low mobility side. This Zeleny interprets as an unresolved negative ion mobility of value about 100 cm./sec. per volt/cm. Such a mobility can only be explained by a carrier that is ion part of its life and electron part of its life, the electronic and ionic phases alternating frequently. The writers indicate that the latter interpretation cannot be correct on the basis of the energies and probabilities involved in electron attachment. Analysis shows that the whole series of the observed phenomena in N_2 by this method are successfully accounted for on the proper application of the simplified theory of electron attachment to the air blast method. Since the attachment phenomena have to date not been applied to this type of mobility study it was felt of importance to present this analysis in order to guide future investigations by this method.

In a recent most interesting article Professor John Zeleny [1] presents among other observations the results of an ion mobility measurement made on negative carriers in relatively pure N_2 gas using his classical air blast method. This nitrogen was contaminated with about 0.3 per cent. of oxygen and consequently there were present a considerable number of free electrons which attached to molecules to make ions in the process of measurement. As a result Zeleny observed for the cases involving longer time intervals two ion mobility peaks, one having a very high mobility 90–100 cm./sec. per volt/cm. or higher, and the other one corresponding to a mobility of the order of magnitude of 3 cm./sec. per volt/cm. He correctly ascribes the first peak of high mobility to an electronic carrier and the other peak to a negative ionic

carrier. Two outstanding features according to Professor Zeleny are of interest. The first is the apparent gradual displacement of the negative ion peak to lower values of the mobility as the measurements are made over longer and longer time intervals. This is interpreted as a progressive ageing of the ion from a mobility of 3 cm./sec. to one of 2.5 cm./sec. The second feature lies in a distinct asymmetry of the electron peak in the mobility curves compared to the curve for these carriers when the air blast was cut off. This asymmetry represents a broadening of the peak on the downstream, or low mobility, side of the curves. It effectually results in placing the center of the peaks at a mobility of from 90–100 cm./sec. per volt/cm. instead of at about 10,000 cm./sec. per volt/cm. characteristic of free electrons.[2] Zeleny thus concludes that this broadening represents an unresolved ion group whose mobility in the one case where it is most clearly defined lies at 90 cm./sec. It is however stated in the paper that the apparent shift is not very definite or regular in its behavior in most cases. To explain the existence of a mobility of such a value Zeleny is forced to consider a negative ion in which the electron is free for a portion of its path, attaching to a molecule to form an ion, detaching itself and reattaching itself many times. In this fashion the distance between electrodes is covered partly as a very mobile free electron and partly as an ion, the average mobility being 90 cm./sec. per volt/cm. This picture is a more modern aspect of the former German concept of the "Zwitter Ion," in which the charge moves from molecule to molecule as the ion moves in the field.

As a result of the investigations of one of the writers and his pupils [3, 6, 7] on the problem of negative ion formation by electron attachment the writers are inclined to believe that the interpretation given by Zeleny concerning the phenomena he has observed cannot be correct. The reason lies in the fact that the mechanism of electron attachment is a complicated phenomenon requiring the dissipation of considerable electron energy, a process which it is very difficult for an elastically reflected electron of low energy to achieve. This fact is borne out by the very low attachment probabilities in oxygen [2, 3] (of the order of 10^{-4} or less). Furthermore the

results of Cravath,[3] indicating that the ionization potential of a negative ion in air (i.e., energy to remove the electron from the negative ion) lies in the neighborhood of 1 volt, again make it most improbable that an electron can detach and reattach itself to even single molecules as frequently as Zeleny is led to assume in order to give a mobility of the value assumed. In fact there has been in electron attachment work no evidence of detachment of an electron from a negative ion below energies of the order of a volt. This belief on the part of the writers led them to consider the possibility of explaining the whole series of published mobility curves observed by Zeleny in N_2 on the basis of the simple electron attachment theory. The resulting analysis established the fact that not only the apparent movement of the negative ion peak to lower mobilities but also the asymmetry of the electron peak could be accounted for by this means. This approach furthermore gives not only the qualitative aspects of the phenomenon but the value of the attachment coefficient in Zeleny's N_2 based on its oxygen content gives nearly the correct scale factor in the equations obtained, a surprising fact when one considers the approximations made for the sake of simplicity. It is the writers' purpose in what follows to indicate how this analysis may be applied to the air blast method in order to interpret these results.

While the exact mechanism of electron attachment is at present somewhat the subject of controversy, the problem has in its broad outline been quite successfully formulated by J. J. Thomson.[5] Thomson assumes that electron attachment is a purely chance phenomenon for an electron and a given molecular species. This phenomenon is governed by an exponential equation involving a probability of attachment which is characteristic of the chemical nature of the molecules involved. On this assumption one of the writers[6, 7, 8] applied the theory to experimental measurements using an alternating current method with photoelectrons. While the results were far from completely quantitatively satisfactory they served to confirm the theory quantitatively at least in order of magnitude.[8] Later more detailed study by Bailey,[9] and by Cravath[3] in the writer's laboratory, using different methods confirmed the general conclusions and verified the

figures in order of magnitude. Both workers found however a variation of the probability of attachment with electron energy [8, 9] and Cravath [3] also observed a pressure dependence. For a very refined analysis of the problem in hand the more accurate data of Cravath and Bailey should be used. However, the ionizing source used by Zeleny precludes any very accurate comparison of theory and experiment so that it suffices to apply the simple theory of attachment as given in Loeb's "Kinetic Theory of Gases" [4] to this problem.

In this text it is shown that the number dy out of y electrons at any point in a gas which attach in moving a distance dx_1 in an electrical field X is given by the relation

$$dy = - hy \frac{\bar{c}_2}{\lambda K_e X} dx_1.$$

Here h is the probability of attachment, a characteristic function for each molecular species. $\bar{c}_2$ is the average velocity of thermal agitation of the electron, λ is the electron free path, K_e is the electron mobility and X is the field strength driving the electrons along the x_1 direction at the particular x_1 in question. This equation may be transformed into a form directly applicable to computation by the relation

$$\frac{\bar{c}_2}{\lambda} = \frac{0.75e}{mK_e},$$

where e is the electron and m is its mass. Whence

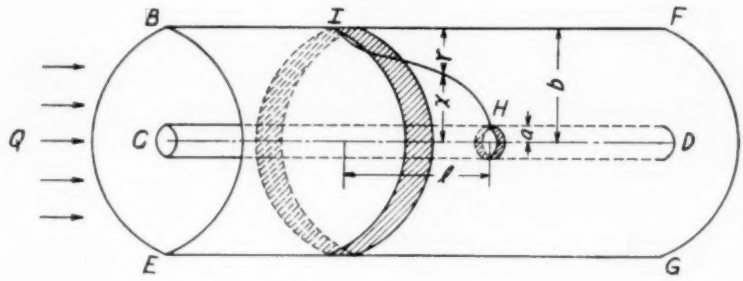
$$dy = - y \frac{0.75eh}{mK_e^2 X} dx_1.$$

One may now apply this equation to the problem of the air blast method of mobility measurement. In Fig. 1 *BFGE* is the outer cylinder of the cylindrical condenser system of radius b. *CD* is the inner cylinder of this condenser system and has a radius a. I is an ideal narrow belt source of electrons, e.g., photoelectrons (these replace the circular ring of holes admitting ionized gas from outside the cylinder which in Zeleny's experiment was ionized by Po). At H on the central electrode is the insulated collecting electrode for the carriers whose downstream distance l from the plane of

I perpendicular to CD can be varied and measured. H is connected to an electrometer to measure the current received for various values of l with a given gas stream velocity and a fixed potential V between $BFGE$ and CD. The curved line marks the path of the carriers. At any point on this path we can call r the radial distance of the ion or electron from I on the outer wall or point of origin, and x the radial distance of this point from the axis. Thus $x = b - r$, and $dx = -dr$. In this condenser the field at any point x from the axis is given by

$$X = \frac{V}{x \log b/a}.$$

FIG. 1.



Now in the expression for dy above, the increment of distance dx_1 used is in reality dr the increment in r, the radial distance from the outer cylinder as regards electron travel. Hence the dx defined by the Zeleny apparatus is $-dr$, or the dx_1 used in Loeb's original expression given above. Hence as applied to the movement of an electron starting at I in the Zeleny tube we may write that in $dx = -dr$ cm.

$$dy = y \frac{0.75eh}{mK_e^2} \frac{\log b/a}{V} xdx.$$

Designate by A the term

$$A = \frac{0.75eh}{mK_e^2} \frac{\log b/a}{V};$$

then $dy = Ayxdx$. Let $y = y_0$, the initial number of electrons liberated at the outer cylinder at I, when $r = 0$ or $x = b$,

and let $y = y$ be the number of electrons left after the electrons have gone a distance r from the cylinder of radius b, or arrived at a point x cm. from the axis.

Then

$$\log \frac{y}{y_0} = -\frac{A}{2}(b^2 - x^2)$$

and we can write

$$(1)\quad y = y_0 e^{-\frac{A}{2}(b^2 - x^2)}$$

and

$$(2)\quad dy = y_0 A x e^{-\frac{A}{2}(b^2 - x^2)}\, dx.$$

Equation (1) represents the number of free electrons remaining after a distance x has been traversed. If $x = a$, y gives the number of *electrons* collected by H, and $y_0(1 - e^{-\frac{A}{2}(b^2 - x^2)})$ represents the number of electrons which have formed ions. These ions are distributed all along the distance x from b to a, and the number of ions formed at any interval dx at a point x is given at once by (2). It is clear that within the regime of applicability of these equations the equations (1) and (2) should enable the calculation of the distributions of the ions and electrons as a function of x.

It now happens that the value of x at which attachment takes place at once fixes the apparent mobility and the downstream distance l at which the ion is caught, of course assuming a narrow electron source. This is clear from the equation expressing ion mobility in the Zeleny method, which reads

$$k = \frac{Q \log b/a}{2\pi V l},$$

where Q is the quantity of gas flowing through the tube per second, or for uniform flow the velocity of the gas multiplied by the cross sectional area, and the other quantities have been defined. Now with electrons starting at I we have carriers of a high speed (10^4 cm./sec. per volt/cm.) going a distance r, or x from the axis, and attaching. The time taken to go this far as an electron in the field is negligible. In going from x to H at $x = a$ the carrier goes as an ion assumed to be of unique mobility for convenience. Conse-

quently while the real mobility of the ion is given by

$$k = \frac{Q \log x/a}{2\pi Vl} \text{ (for the ion really starts from } x),$$

it has been assumed to start from b and its computed mobility from its observed downstream distance will be given by

$$k_{\text{comp.}} = \frac{Q \log b/a}{2\pi Vl}.$$

The l here is the observed downstream distance which corresponds to an ion that started from x. Hence the apparent mobility for an electron which attached at x and was caught at l will be related to its real mobility by the expression

$$k_{\text{comp.}} = \frac{\log b/a}{\log x/a} k.$$

Further knowing its real mobility k the downstream distance l, at which such an ion formed at x reaches H at $x = a$, will be found at once from

$$k = \frac{Q \log x/a}{2\pi Vl}.$$

It is thus at once possible to plot the fractional number of carriers starting from I which reach each point l downstream on CD and have a mobility $k_{\text{comp.}}$. To do this it is merely essential to evaluate A for a given gas and assign values to A, V, b, a, h, and K_e.

The practical application of this analysis to Zeleny's measurements is however complicated by one difficulty. The band of electrons from his source (equivalent to I) has a finite width due to electron diffusion which is not inconsiderable. These give rise to a distribution of ions or electrons in the form of a bell shaped peak of considerable extent along l. To solve the distribution problem with such a finite width of beam requires that one assign to the electron peak an area equivalent to equation (1) with a finite extension along l evaluated for $x = a$, and to each point downstream a similarly shaped peak of area equal to the value given by equation (2). These overlapping distributions must then be

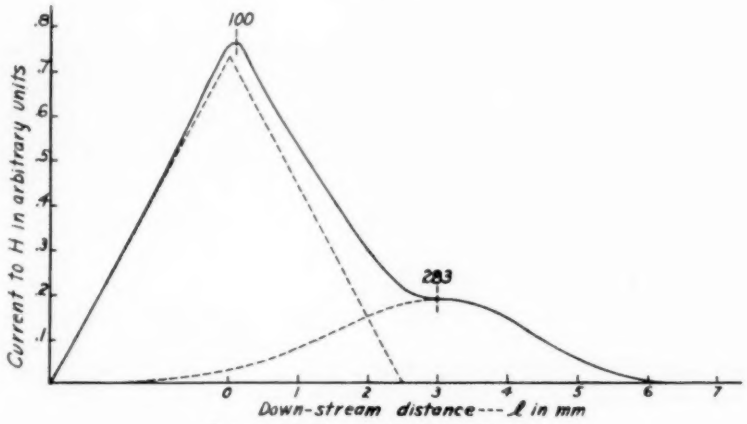
added up to give the resultant distribution of ions and electrons along l. To facilitate computation the peaks were in this work chosen as triangles whose base was taken equal to the undisturbed electron peaks observed by Zeleny in the absence of an air blast, and the area was then adjusted for each triangular peak by altering its altitude in conformity with equation (2). It was also found that when summing up the peaks represented in amplitude by equation (2) it was easier to take the peaks at 19 equally spaced intermediate values of x corresponding to 19 different values of l. These ranged from $l = 0$ to $l = 16$ mm. in one case and 4 mm. in the other. The corresponding values of k ranged from $k \cdot = . \infty$ to $k = 2.2$, the assumed negative mobility. To facilitate calculation in using these peaks the amplitudes were selected by actually choosing the differences in the values of y for 20 equally spaced values of x instead of attempting to adjust equation (2) using appropriate values of dx. This procedure is logical since the changes in y between any two values of x and hence l represent the actual ions formed in the interval. The use of the quantity Q was obviated in these calculations by using the observed values taken direct from Zeleny's curves for nearly equivalent conditions.

With the above discussion it is only necessary to include the data for evaluating A in order to complete the description of the computation.

The value of h for the O_2 in air at atmospheric pressure in the low fields here used has been estimated as 2.5×10^{-5}.[2, 6, 8] Zeleny states that his N_2 had about 0.3 per cent. of O_2 present. On the basis of O_2 content alone the value of h for his gas should be 7.5×10^{-8}, $e = 4.77 \times 10^{-10}$ E.S.U., $m = 8.99 \times 10^{-28}$ grams, $b = 7.3$ cm., $a = 2.5$ cm.[10] For the two curves computed the values of V chosen were 30 volts or 0.1 E.S.U. and $V = 90$ volts or 0.3 E.S.U. The value used for K_e was 10^4 cm./sec. per volt/cm., *K_e being for simplicity assumed constant.*[2] This procedure is not strictly accurate for K_e is a function of X and hence of x. The use of a constant K_e for calculations of this kind in the past, where qualitative agreement is sought has not been fraught with serious consequences.[6, 8] Thus in view of the considerable complexity introduced by, and the uncertainty of the exact

variation of K_e in this region of pressure and field strength a constant value was chosen. Thus $K_e = 10^4$ cm./sec. per volt/cm. $= 3 \times 10^6$ cm./sec. per E.S.U./cm. Using the two values of V, A came out 0.036, and 0.010; for convenience in calculation the values 0.04 and 0.0133 were actually used. Twenty equally spaced values of x were then chosen from 2.5 to 7.3 in increments of 0.25 cm., and the values of l, the downstream distance, computed from them, using the actual values given by Zeleny in his paper. The result of these calculations are illustrated in Figs. 3 and 2, for values of V of 30 and 90 volts. For comparison three curves of

FIG. 2.

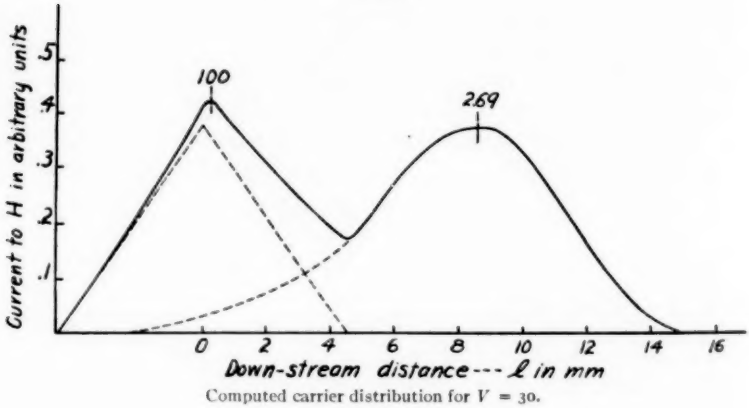


Computed carrier distribution for $V = 90$.

Zeleny obtained at 50, 30 and 10 volts are shown on a similar scale in Figs. 4, 5 and 6. The values chosen for computation were selected so as to be in the same range as those given by Zeleny. It is seen clearly that Fig. 3 represents a stage of development of the curves intermediate between that of Figs. 5 and 6, while Fig. 2 corresponds closely to the picture given by Fig. 4. In Figs. 2 and 3 the full curves are the calculated curves while the dotted curves indicate the original triangular electron distribution chosen, of magnitude indicated by equation (1) and the extrapolation of the ion curve which added to the triangular peak gives the unsymmetrical form. In spite of the dissimilarity in the initial distribution

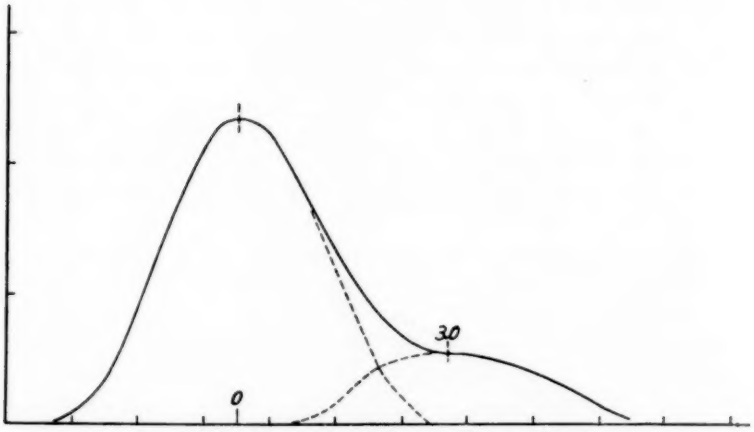
used and the real curves (triangle versus bell shaped curve) the unsymmetry of the peaks and the apparent displacement of the negative ion mobility with age to lower values is quite

FIG. 3.



Computed carrier distribution for $V = 30$.

FIG. 4.

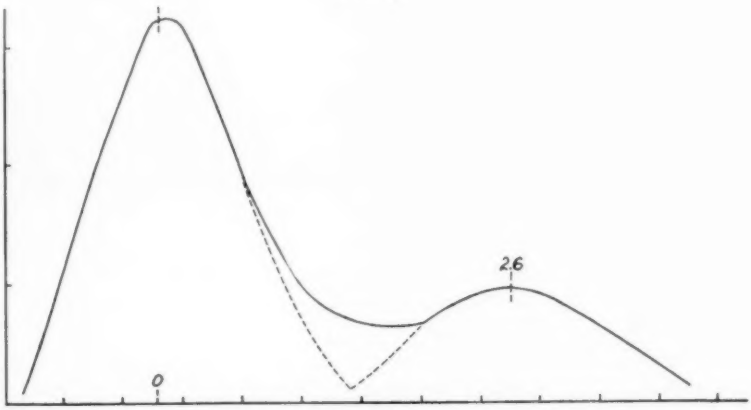


Zeleny's observed distribution curve for 50 volts.

conspicuous. With broader peaks the dissymmetry of the negative peaks would have been more pronounced. Also the increasing width of the experimental distribution curves with downstream distance l due to greater diffusion in longer time

intervals in practice leads to a greater apparent shift in the negative ion peak with age than that given by the computed curves which did not take such a broadening into account. If one consider therefore the rather great departure of Zeleny's real experimental conditions (i.e., increased spreading by diffusion with age, and ageing and attachment in the ionization space outside of the cylinder *BFGE*), the close duplication of the phenomena using values of h computed on the basis of the oxygen content given with no attempt at adjustment of constants, makes the agreement sufficiently striking to

FIG. 5.



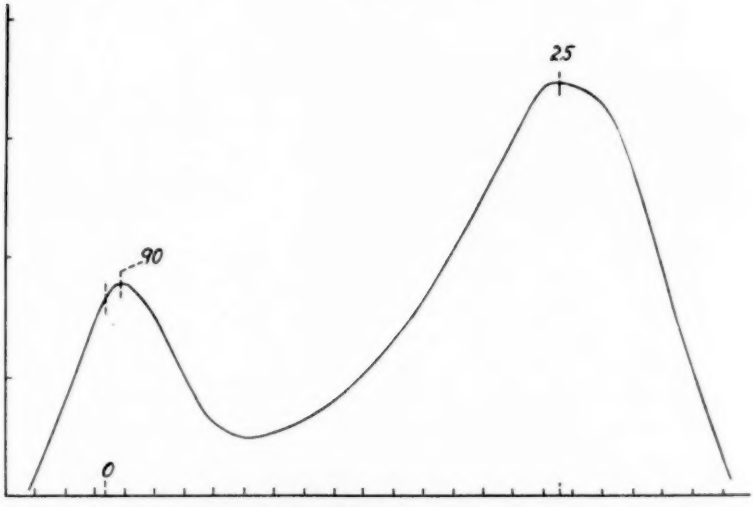
Zeleny's observed distribution curve for 30 volts.

cause one to feel sure that the interpretation given is essentially correct. In addition the equations show that for higher and lower potentials one could closely reproduce the other curves observed by Zeleny.

That Figs. 2 and 4 agree for potentials of 90 and 50 volts and that the Fig. 3 at 30 volts lies between the 30 and 10 volt curves of Figs. 5 and 6 need not cause any concern. This could be accounted for by several of the neglected factors such as the relatively small variation of K_e with the small variations of X and of the variations of h with X. It might also indicate merely that too high a value of h was taken such that the chosen value of V had to be about twice as high as observed in order to give a close agreement.

It is easily seen from these curves that the unsymmetry of the electron peak is due merely to the appreciable attachment and ion formation taking place in the weak field at the outer part of the cylinder. The variable initial state of the attachments for the carriers coming through the holes with different air current velocities in Zeleny's measurements, together with the natural width and shape of the peaks and the unsymmetry produced by attachment, as well as the crowding of mobility values at these peaks readily account for the unsatisfactory values observed by Zeleny for his

FIG. 6.



Zeleny's observed distribution curve for 10 volts.

assumed fast ions of mobility about 100 cm./sec. per volt/cm. Thus the curves of Figs. 2 and 3 show that while the unsymmetry is real it is entirely due to an expected and calculable phenomenon and the assumption of the real existence of ion mobilities of the order of 100 cm./sec. is unnecessary.

It is further seen that the question of the change of mobility of the negative ions with age as indicated by the apparent migration of the ion mobility curves of Zeleny downstream is clearly solved by the analysis above. In the first place in such diffuse and interconnected curves as

Zeleny obtained, together with the peculiar mobility scale for downstream distance it is almost impossible to fix the actual peak value of the curve and thus get its downstream distance, to say nothing of fixing a satisfactory mobility to the peak as these change so rapidly with distance on the upstream side. Secondly the actual peak will appear gradually to migrate downstream as attachment proceeds from the nature of the equations which are based on the assumption of a unique ion mobility. Finally the broadening of the distribution curves with downstream distance (which were not included in the calculations) further increases the rate of apparent downstream movement of the ion peak on ageing. It is therefore clear that no importance at all can be ascribed to this phenomenon which really follows directly from theory and the finite width of the ion streams.

It would be indeed rash to assert that the negative ion in N_2 does not actually alter its value with age. In fact one of the writers has definite evidence to the contrary in the case of Na^+ ions in relatively pure N_2 where mobilities of at least two other ions were observed beyond the single Na^+ ion.[11] This ageing, however, took place for positive ions in shorter time intervals than those involved in Zeleny's investigation on negative ions. The only statement that can be made in this respect is that there is no evidence from *these experiments* that negative ions change mobility with age. In fact the statistical process of electron attachment is such that it is almost hopeless to consider the possibility of making a critical study of negative ion mobilities in the presence of free electrons which are attaching. Whether in fact in gases like O_2, N_2, H_2, etc., negative ions undergo ageing and change of mobility at all is an open question. The only conclusion, therefore, to be drawn is that the apparent movement of Zeleny's negative ion peaks furnish no information on the subject, being due to the attachment process.

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Oct. 26, 1931.

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LAMINAR AND TURBULENT FLOW ABOUT SHIP MODELS.

BY

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1. SUMMARY.

Total fluid resistance of a ship or ship model is commonly separated into viscous and dynamic components, corresponding to the tangential and normal components of the forces acting on the boundary surface. This separation has hitherto depended on calculated values of the viscous resistance based on experiments in which dynamic resistance was reduced to a minimum and assumed zero. Irregularities in viscous resistance are known to exist, however. There are two different regimes of flow in which tangential resistances have radically different values and it so happens that ordinary lengths and speeds of ship models bring them near, if not within the range of transition from one regime to the other. On the other hand, the dynamic resistance is still more obscurely related to the elements of geometrical form and speed than is the viscous resistance, and its systematic analysis continues to rest very largely on an empirical basis.

The problem is to obtain accurate data as to these two quantities, such as will serve as an adequate basis for theoretical considerations, the only quantity amenable to experiment being their sum. Experimental methods have been refined considerably without bringing a definitive solution. A novel point of view is herewith presented which, it is hoped, may ultimately lead in the desired direction.

2. PROCEDURE IN ANALYSIS OF RESISTANCE CURVES.

The adoption of a fixed expression for viscous resistance places the whole burden of uncertainty due to instability of flow as well as that of experimental error on the dynamic residuum. It is now proposed to divide this load of error by

introducing a more flexible assumption as to the viscous resistance; to avoid making the problem completely indeterminate it is then necessary to introduce some restrictive assumptions as to dynamic resistance.

The assumptions now made are as follows:

In a series of geometrically similar models, specific dynamic resistance is a function only of Froude's number, speed divided by square root of length. Specific viscous resistance in the laminar regime or the turbulent regime is a function of Reynolds' number, proportional to the product of speed and length, but the transition from one regime to the other follows different paths under different circumstances; these may be specified by reference to a single parameter.* A fixed expression for specific viscous resistance in each of the two regimes is adopted, leaving the desired flexibility to be obtained in choice of the transition parameter.

An empirical expression for dynamic resistance is used in which adjustment of three constants is made to suit observations on each model. Averages of these constants from the whole series of models are then taken and dynamic resistance of each model calculated by use of the average values of the constants.

Combining the dynamic resistance so determined with the fixed expression for turbulent friction gives a curve of total resistance. The observed data on total resistance lie below this curve, and at small lengths and low speeds they approach the curve for laminar friction.

Circumstances affecting the stability of regime are at least partly controlled. With fair consistency the observed spots lie on a transition curve specified by a single value of the parameter so long as these circumstances are unaltered.

Correlations of the circumstances affecting stability with the values of the transition parameter are sought.

3. RESULTS OBTAINED.

Plotted data for nine similar models ranging in length from forty feet to two feet are shown. C designates the transition parameter. In each case the upper curve, for which $C = 0$, gives the total resistance in the turbulent

* See Appendix B.

regime. For the shorter models a curve lying below and to the left gives total resistance in the laminar regime. Observed values are plotted, with transition curves corresponding to noted values of C.

The turbulent friction curve is the same for all models in this series. As length diminishes, the maxima of dynamic resistance, which are at definite values of Froude's number, come at progressively lower values of Reynolds' number. The separation of dynamic resistance from frictional resistance may be said to be accomplished by sliding the dynamic resistance curve into different positions on the turbulent friction curve.

Beginning with the longest model, $L = 40$ feet, it will be noted that of the characteristic maxima due to dynamic resistance only one lies at a speed within the limits used. The dynamic resistance below $VL = 150$ is negligible, and the spots agree quite exactly with calculated values. Above $VL = 200$ the effects of interference of walls and bottom with flow past the model cause the observed values to fall above the curve. This model is twice the length for which the basin is designed.

At $L = 30$ feet the agreement between calculated and observed data is close at all speeds except the highest, where the restricted channel again produces an observed value that is too high.

The agreement between calculated and observed data extends with the 20 foot model down to $VL = 70$. Beyond that the observed spots fall away from the calculated curve by an amount which increases as VL diminishes. Four series of spots are shown here. In that marked 28F a fin was towed ahead of the model so as to leave a turbulent wake. The resulting forward motion of the water, if appreciable, would have reduced the model resistance, instead of which these spots lie closely on or a little above calculated values. The series marked 28B falls a little below the curve, and that marked 23, except for one spot, further below. A transition curve, $C = 1{,}800$, is sketched in, and the spots group themselves about it, but are more widely scattered than in the case of the 30 foot model. This scattering is a characteristic feature of resistance in the transition range. The last series,

marked WD, was run in very still water down to very low speeds.

This sagging of observed resistances below the values based on the curve for turbulent friction is due to the laminar flow about the bow. As the water streams past the model the breakdown into the turbulent regime does not occur until a certain distance aft of the stem is reached. Resistance in the laminar regime being less than in the turbulent regime, the total resistance is reduced by this effect. The amount of the reduction is greater at lower speeds due to the breakdown occurring further aft, and in smaller models due to the area under the laminar regime forming a greater part of the whole.

Each of the next three models was run up to high relative speeds, at which dynamic effects not contemplated in the calculations cause the curves to drop below observed values. This effect is consistently shown by all three models. It occurs at values of VL well above those with which we are concerned in studying the transition from one regime to the other. The significant point about these high speeds is not this rather small discrepancy, but the fact that a last maximum exists beyond which at higher speeds the specific resistance continues to drop away. In the case of the 14 foot and 10 foot models calculated resistance agrees with observation out well past this last maximum. The scattering of the spots in this range for the 5 foot model is due to the small total resistances, the model having only 1/4 the length and about 1/64 the total resistance for which the dynamometer was designed.

On the 14 foot model the dropping away of observed values at lower speeds begins at about the same value of VL as on the 20 foot model.

There begins to be enough separation between the different series to justify separate values of the transition parameter. Thus the series 28F, run in turbulent water, falls slightly below the curve $C = 0$; the effect of the turbulence cannot quite make up for the forward flow of water caused by the fin, but the correction is made by a moderate value of the parameter, $C = 700$. The WD series and the 28B series agree rather well at $C = 2100$, but the 23 series falls somewhat higher, about $C = 1500$.

In the 10 foot model the 28F series coincides well with the calculated curve except that the two spots at low speeds fall high. Series 28B and 23 are in fair agreement on a parameter value $C = 1500$. The WD series shows C about 3000, but at the lowest speeds the resistance curve turns upward. This marks the end of the transition, the length over which flow is laminar having now approached the full length of the model. The characteristic feature of laminar flow is inverse proportionality of specific resistance with square root of speed times length. The curve approaches a slope corresponding to this condition, but does not reach it. The value of the constant of proportionality at the last spot is 1550, compared with Prandtl's value of 950.

In the case of the 5 foot model the lower speeds were all run in a small basin. Below $VL = 1$ the flow is distinctly all laminar, and the value of 1150 for the constant approaches Prandtl's value of 950. Ten series were run on this model, but only those in the transition range call for special comment. Series 21W was run with a fine wire wrapped around the bow. The transition is well over toward low values of VL, the parameter being about 800. Series 18W was made under the same conditions at a different time, and fits in well with series 21W. Series 7W was also made under the same conditions, but at lower speeds. Departure from the laminar curve is small even at $VL = 2$.

In series 28B the transition lies further toward high VL, but the scattering is such that no parameter was evaluated for this series.

Series 21B and 18B fit in well together on a parameter value of about 2200.

The maximum difference in total resistance of this model, due to an almost infinitesimal roughening of the bow, occurs about $VL = 4$ and amounts to an increase of about 70 per cent. over the lower value.

At still smaller lengths confirmation of the systematic variations already pointed out is obtained. Somewhat lower values of the laminar constant are obtained, but none as low as Prandtl's value. On the other hand, there are some indications that the transition curve joins the laminar curve less abruptly than the formula would indicate; it is possible

that pure laminar flow is not obtained until a value of VL of the order of 0.1, at which the constant might actually have the value 950. Transition parameters are evaluated as shown in Table I.

TABLE I.

Model Number.	Length in Feet.	Run.	Transition Parameter.
2841	2	28W	700
		28B	
2604	2.5	28W	600
		28B	2400
		24	2400
2842	4	28W	1500
		28B	2600
			1900
2605	5	28F	
		18W	800
		21W	800
		7W	
		28B	
		18B	2200
		21B	2200
		24	
		26	
2509	10	28F	0
		28B	1500
		23	1500
		WD	3000
2505	14	28F	700
		28B	2100
		23	1500
		WD	2100
2501	20	28F	
		28B	1800
		23	1800
		WD	
2513	30		
2536	40		

Note: B means bare hull. W means wire on bow. F means fin ahead. WD means weight dynamometer runs at slow speeds in quiet water.

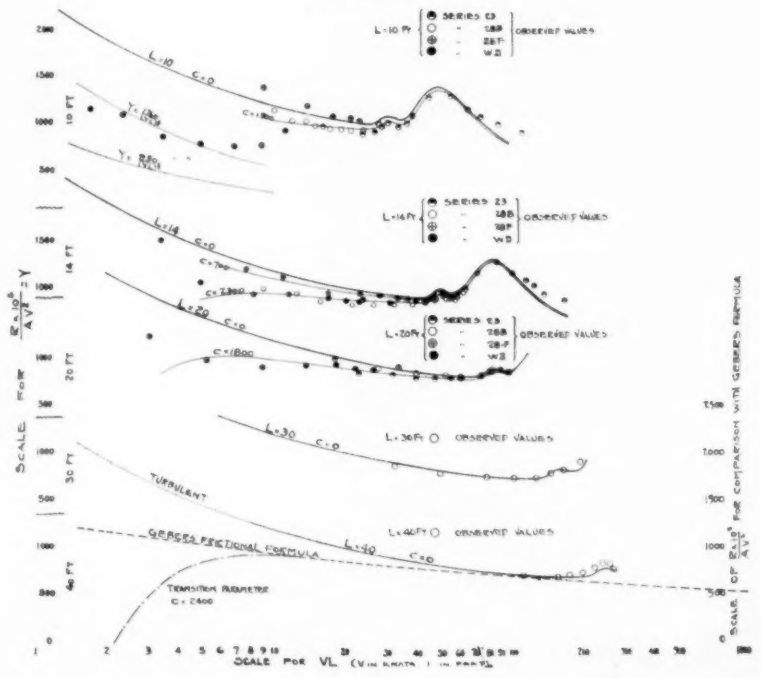
4. COMMENTS AND CONCLUSIONS.

Departures from the curve of turbulent friction due to laminar flow about the bow become appreciable at values of VL approaching 100. For models 20 feet long, this comes

above the middle of the speed range in actual use, leaving speeds lower than 5 knots subject to more or less interference. This limit corresponds to a speed of 22 knots in a 400 foot ship, and 35 knots in an 800 foot ship. It is therefore apparent that interference is practically always present in a 20 foot model.

However there are several mitigating circumstances. The Gebers formula [1] reproduces the transition curve for a parameter value of 2400 within close limits down to $VL = 10$. (See Plate I.) This approximately covers the full range of

PLATE I.

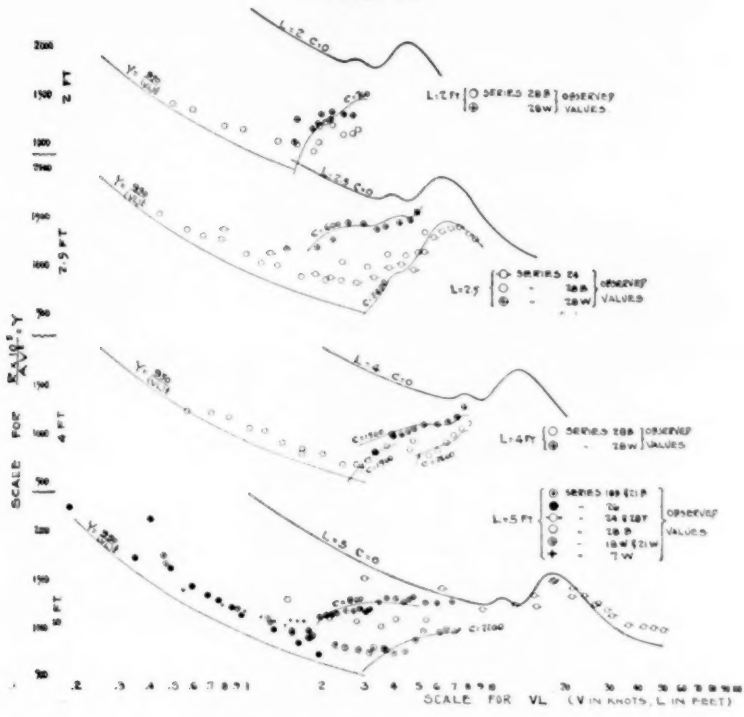


speeds with 20 foot models, and accounts for the success with which the Gebers formula has been used. The original data on which the Gebers formula is based are not correctly given by that formula at low speeds and lengths, and all data from the shortest model used by Gebers, about 4 feet long, were

[1] *Schiffbau*, Vol. **XXII**, 1919.

rejected in evaluating the constants because the values seemed too low. These values approximate the transition curve for $C = 2400$. It therefore appears that while the Gebers formula may be used for models 20 feet long, there will be increasing difficulties as the lengths become less.

PLATE II.



Something can be done to reduce the value of the transition parameter, by artificially stimulating the onset of turbulence about the bow. This happens automatically, for example, as soon as the dynamic resistance reaches the stages at which the bow wave breaks. Perhaps wooden models perform somewhat more satisfactorily in this respect than paraffine models.

It should be noted, however, that reduction of the transition parameter will not improve agreement with Gebers' formula at all speeds.

The most significant feature of the whole study lies in the indication of the sensitiveness of frictional resistance at certain speeds and lengths to slight modifications in conditions which have not always been successfully controlled. Even though Gebers' formula be approximately correct at $C = 2400$, there has been no positive method of holding the transition parameter at that value. The close grouping of observed spots about a curve drawn for a fixed value of the transition parameter shows that changes in the value of the parameter due, for example, to changes in surface conditions, are rather slow. If the parameter could be evaluated by runs of high precision at low speed, and the procedure in further work regulated with a view to eliminating changes in the parameter, it is thought possible that improved precision in model tests would result from use of a friction curve based on the specific value of the parameter so determined.

The values of the constants used in calculation of dynamical resistance are shown in Table II. The various

TABLE II.

Constants for Dynamic Resistance.

m	1	2	3	4
C_m	577	143	46	26
n	2.92	8.17	4.77	5.48
V_m:				
L = 40	10.37	6.24	4.87	3.54
= 30	8.98	5.40	4.22	3.07
= 20	7.32	4.41	3.45	2.51
= 14	6.13	3.69	2.89	2.09
= 10	5.19	3.12	2.44	1.77
= 5	3.66	2.21	1.72	1.25
= 4	3.28	1.97	1.54	1.12
= 2.5	2.59	1.56	1.22	0.89
= 2	2.32	1.39	1.09	0.79
$\frac{V_m}{\sqrt{VL}}$	1.64	.99	.77	.56

uncertainties involved led to values for these constants, as taken from each model separately, which varied somewhat from model to model. With three constants to deal with, it is possible to obtain more than one set of constants which will give a curve fitting the observed data within limits of observational errors. In some cases the average values, taken from all the models, do not fit quite as well as the values

found for the specific model. But the important fact is that a single set of constants does exist which will serve for the whole series of models, within reasonable tolerances. This fact constitutes the principal logical justification for the whole process by which dynamic resistance is here reduced to a formula.

The values of these three constants would depend strongly on the underbody form of the models. The form actually used was quite narrow and deep, designed to give maximum wetted surface with minimum displacement obtainable in a form with enough buoyant stability to permit towing without constraints to hold the model in the upright position.

This form is the only one for which these constants have been evaluated. When this analysis has been extended to a variety of other forms, and correlations established between the constants and the elements of underbody form, it will be possible to obtain dynamical resistance of new forms by calculation without necessity for model tests.

APPENDIX A.

Historical Summary and References.

Ship resistance was studied by Newton,[2] and extensive data were taken by Beaufoy[3] in 1793 to 1798. Successful separation of viscous and dynamic effects was first made by Froude.[4] The discovery that two distinctive regimes of flow exist was made by Reynolds[5] whose work dealt, however, only with flow in pipes and channels. A recent extended survey of the subject from this point of view is given by Schiller.[6]

Application of Reynolds' ideas to the ship problem was deferred until after 1900, but papers on the subject have appeared recently with increasing frequency. These are well summarized in three recent German Handbücher.[7]

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[2] *Principia*, Prop. 36 and those following.

[3] *Nautical Experiments*, London, 1834.

[4] British Association Report, 1872 and 1874.

[5] *Phil. Trans. Royal Society*, 1883.

[6] *Physikalische Zeitschrift*, 1925, p. 566.

[7] *Handbuch der Physik*, Vol. **VII**, edited by Grammel. *Handbuch der Mechanik*, Vol. **V**, edited by Auerbach. *Handbuch der Experimental Physik*, Vol. **IV**, edited by Wien and Harms.

APPENDIX B.

Formulæ, Units, and Coefficients.

All data are in terms of specific resistance, which is resisting force divided by area of wetted surface and by square of speed at which the model was towed. This quantity in general is denoted by y, and as plotted is in practical units, R in pounds weight times 10^{-5}, A in square feet, and V in knots of 6080 feet per hour. If absolute units be used, and the expression divided by density of the fluid, a dimensionless coefficient of resistance is obtained which is in common use in some laboratories. To convert the specific resistance here plotted into the dimensionless coefficient, divide by 5·54. The quantity C_f used in German work on the subject is twice as great, making the conversion factor 2.77.

Reynolds' number is proportional to a speed times a length only so long as viscosity of the fluid remains unaltered. The tests were made in fresh water at a mean temperature of 65° F., for which VL as plotted must be multiplied by 150,000 to obtain Reynolds' number as a dimensionless ratio.

Laminar resistance is considered to follow the accepted expression

$$y = \frac{\text{const.}}{\sqrt{VL}}.$$

Prandtl's value of this constant is 950×10^{-5}. In the 10 foot model, a curve plotted from a higher value more nearly approximating the actual data is also shown. The discrepancy generally becomes smaller at smaller lengths. There is a suggestion in the trend of the spots at low values of VL that the transition to pure laminar flow may still be incomplete at the lowest values obtained, and that Prandtl's value would be approached still more closely at still lower VL.

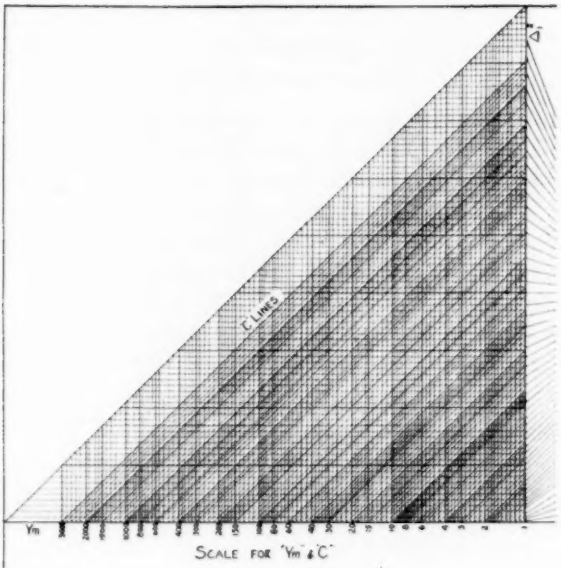
Turbulent resistance according to Prandtl [8] is

$$y = \frac{.0184}{(VL)^{1/5}}.$$

Whatever theoretical backing this type of friction formula may have, it has the fatal defect of making specific resistance

[8] Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, Dritte Lieferung, 1927.

PLATE III.



CONSTRUCTION AND REPAIR DEPT.
NAVY YARD, WASHINGTON, D.C.

GRAPHICAL SOLUTION OF EQUATION

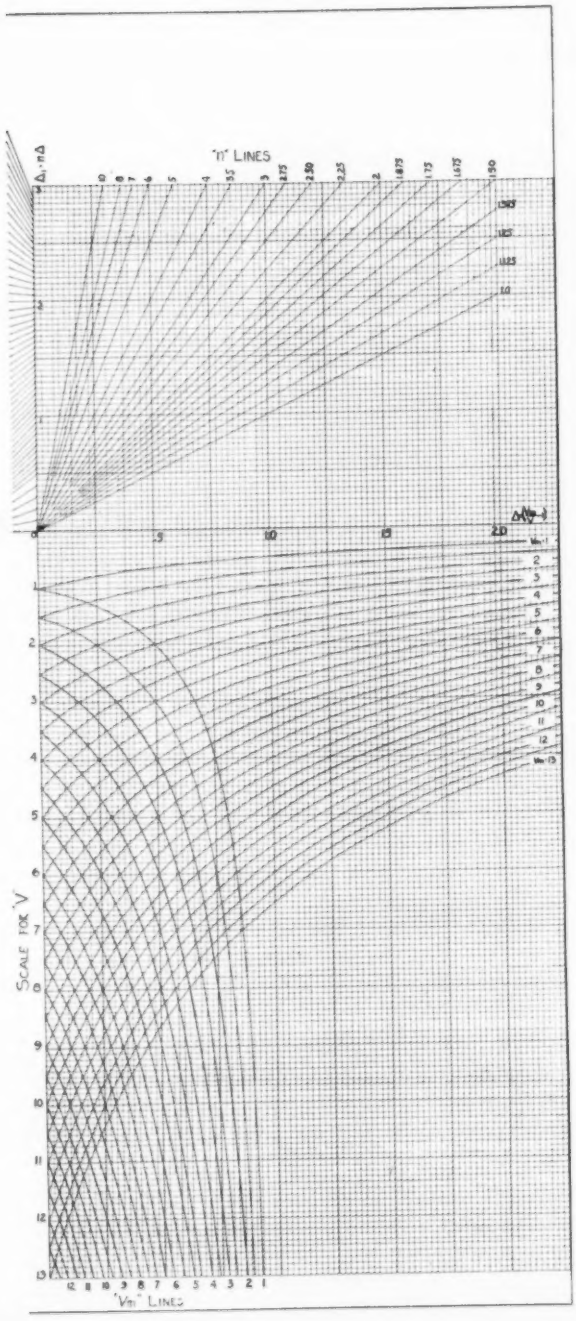
$$Y_m = Ce^{-n^2\left[\left(\frac{Y_m}{V}\right)-1\right]^2}$$

METHOD OF SOLUTION

FIND Δ FOR GIVEN VALUES OF V AND V_m AND TRACE TO PROPER 'n' LINE LOCATING Δ_1, THENCE TO Δ_1^2 FOR THIS VALUE OF Δ_1^2 AND THE GIVEN C READ Y_m AS ABSCISSA.

FEB 1934

PLATE III.



approach zero at large values of VL. At ship speeds and lengths ($VL \doteq 25000$) the frictional resistance is known to be greater than this formula indicates. Von Karman [9] suggests that the formula is really based on empirical data; this being true it is not surprising that it should fall short of perfect agreement after a 50-fold extrapolation. Within short ranges of VL the formula [10]

$$y = a + \frac{b}{(VL)^n}$$

will equally well represent any data hitherto available, and over long ranges will do much better. The plotted values of turbulent frictional resistance were obtained from the formula

$$y = 420 + \frac{2000}{(VL)^{2/5}}.$$

The transition curve is obtained by subtracting from the curve of turbulent resistance a function of VL which diminishes as VL increases. Prandtl [8] uses a correction of the form

$$y = \frac{\text{const.}}{VL}$$

as a first approximation. At short lengths it is found impossible to adjust the single constant to fit the observed form of the transition curve, which falls away too steeply toward lower values. Better agreement with observation is given by the expression

$$y = \frac{\text{const.}}{VL - 1}$$

and the transition constants referred to are evaluated on this basis throughout. The constant is here adopted as the transition parameter and denoted by C. Prandtl evaluates it at 3140×10^{-5}, but the present data indicate a variety of values.

Dynamic resistance is well known to follow a curve with a series of maxima. It has usually been supposed to be a

[9] Abhandlungen aus dem Aerodynamischen Institut, Aachen, Heft 1, p. 6.

[10] *Transactions Society Naval Architects and Marine Engineers*, 1929, p. 49.

curve of generalized parabolic form, perhaps with increasing exponent at higher speeds, with superposed fluctuations due to interference of the wave systems generated at the bow and stern. This supposition is contradicted, however, by the fact that in some cases, like the present one, at very high speeds, $V > 2\sqrt{L}$, total specific resistance is observed to fall off with increasing speed. Good agreement with the data is obtained from the purely empirical assumption that the curve is formed by superposing a series of curves, each of the form

$$y_m = C_m \cdot e^{-n^2\left(\frac{Vm}{V} - 1\right)^2}.$$

m denotes the order, V_m being the speed at which the maximum of this order occurs, C_m the value of y_m when $V = V_m$, and n a coefficient governing the width of the peak. For convenience a graphical solution of the equation was prepared and is reproduced in Plate III.

Frozen Pack Fruit. (U. S. Department of Agriculture, Clip Sheet No. 706.) Pacific coast strawberries, packed and frozen for more than a year in a laboratory experiment, have maintained excellent quality and from 90 to 99 per cent. of the organisms in them that cause spoilage have been killed, the United States Department of Agriculture reports. Effective killing of these organisms is one of the chief problems in the frozen-pack method.

Microbiological studies on frozen pack fruits and vegetables were started last year in the recently established frozen-pack laboratory of the Bureau of Plant Industry in Seattle, Wash. The work has been under the direction of James A. Berry, and preliminary results are now available.

Sound fruit, suitably packed and promptly frozen, was excellent in quality after being frozen for 12 months, the report shows. Inferior or overripe fruit, however, had a poor appearance when thawed and was not in marketable condition.

In the strawberry studies, microbiological analyses were made of 100 samples of Pacific Northwest strawberries, chiefly of the Marshall variety. The samples were packed in the 1930 season in paper containers and in No. 2 tin cans, without and with vacuum, and in sirups of different strengths. Fifteen months after the fruit had been stored at 15 degrees F. the fruit was thawed and again analyzed microbiologically. This analysis showed that 90 to 99 per cent. of the organisms had been killed.

Berries having 10,000 organisms per gram at the time of packing gave counts of 1,000 or less. The chief surviving organisms are species of Penicillium and spore-forming bacteria. In particular, the cold killed most of the yeasts, so that cans defrosted and kept in the laboratory two weeks failed to develop pressure. Other things begin equal, sealed cans showed a somewhat higher "kill" than paper containers, which are not air-tight.

R.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

CLOCK-CONTROLLED CONSTANT-FREQUENCY GENERATOR.

In the January number of the Bureau of Standards Journal of Research a motor-generator set is described in which the motor is forced to rotate in synchronism with signals from a standard clock circuit. This result is obtained by first running a specially wound synchronous motor from a 3-phase commercial power line. The field of this synchronous motor is then electrically rotated about the motor frame by an amount which exactly compensates for the departure of the frequency of the commercial power from true 60 cycles. This rotation of the motor field is effected by a rotary synchroscope operated by thyratrons which are in turn controlled by a clock-driven 60-cycle tuning fork.

From instant to instant the motor does not maintain exact synchronism with the tuning fork signals but always hunts slightly about the position of exact synchronism. The probable error introduced into observations by this hunting, when the generator is used as a timing device, has been determined experimentally to be only $\pm$ 0.0003 second. Safety devices are provided which definitely guard against the possibility of this error amounting to as much as $\pm$ 0.004 second. When any appreciable intervals of time are to be measured, this random error becomes negligible and the accuracy of the machine is limited solely by the accuracy of standard chronometer used.

The uses, limitations, and possible improvements of the machine are discussed. Its load capacity is 4 kw., and it is capable of taking a suddenly applied load of 2 kw. without serious hunting.

PRESERVATION OF RECORDS IN LIBRARIES.

Mention has been made of the bureau's survey of libraries, carried out as a part of its researches on the preservation of

* Communicated by the Director.

records (*Technical News Bulletin* No. 175, p. 124; November, 1931). The storage of books in representative public libraries in various parts of the United States was observed with particular reference to such external deteriorating agents as acidic pollution of the atmosphere, high temperature, variation in atmospheric humidity, dust, and light. It was found that while the deteriorative effects of high temperature, dust, and light were generally recognized and adequately guarded against, little attempt was made to rid the atmosphere of acid fumes or to control the humidity of the atmosphere within the limits considered desirable.

Sunlight has long been recognized as extremely deteriorative to papers. Experiments made at the bureau have shown that direct exposure of various types of current record papers to sunlight for only 100 hours on each side, reduced their folding strength to from 25 to 63 per cent. of the original. It was also found that rosin sizing, as used in such papers, darkens readily on exposure to light, thus possibly contributing to their yellowing with age. The book stacks of the public libraries are generally lighted by frosted incandescent bulbs which are turned on as needed, thus the effect of light is quite well guarded against.

Protective treatment of the atmosphere in the libraries was found to be generally confined to removal of dust and rather limited regulation of temperature and humidity to prevent excessive heat, dryness, or moisture; oil filters and water sprays being used for these purposes. Air polluted with sulphur dioxide from the combustion of fuels is probably the most potent external deteriorative agent, as in contact with paper, sulphur dioxide forms sulphuric acid, the well-known corrosive "oil of vitriol." Change in atmospheric humidity concentrates the sulphuric acid in paper, thus hastening its embrittling effect, as does excessively high temperature. The bureau therefore recommends automatic control of temperature at 65–75° F., and of humidity at 45–55 per cent. These are healthful conditions and they can readily be maintained with equipment such as is commonly used in manufacturing plants, office buildings, and theaters.

The need of protection against sulphur dioxide fumes is emphasized by reference to statistics which show, for example,

an annual precipitation of sulphuric acid amounting to 83 tons per square mile in Philadelphia, and 194 tons in Glasgow. The bureau has found that printing and writing papers exposed 10 days to an atmosphere containing 5 to 10 parts of sulphur dioxide per million parts of air had lost as much as 40 per cent. of their original strength. The use of an alkaline water spray is recommended for combating acid in the atmosphere. The survey reported is one phase of a general study of library storage of records being made at the Bureau of Standards with the assistance of a fund granted for the purpose to the National Research Council by the Carnegie Corporation.

The complete report of this work is now available as Bureau of Standards' Miscellaneous Publication No. 128. Copies can be obtained from the Superintendent of Documents, Washington, D. C., for five cents, cash.

VOLUME CHANGES IN METALS DURING CASTING.

The American Foundrymen's Association and the bureau have been coöperating in a study of the shrinkage of metals during casting and the influence of such shrinkage on the production of sound castings. This work will be reported in the January number of the *Bureau of Standards Journal of Research*.

The total shrinkage of a metal or alloy for casting purposes has a profound influence on the foundry practice for that particular metal. It affects the size of the pattern required to give a casting of definite dimensions (pattern maker's shrinkage), the design of a casting, the size, type, and location of risers and feeders needed, the liability of castings to crack in the mold and the soundness of a pressure test.

The present paper describes a method which has been developed for measuring the volume changes undergone by a metal or alloy with change in temperature as it cools from any temperature in the liquid state to ordinary temperatures. It defines as the three types of shrinkage to be considered: (1) Shrinkage of the metal in the liquid state; (2) shrinkage during solidification; and (3) shrinkage in the solid state; and considers in detail the sand-cast cone method for determining

shrinkage during casting and the more fundamental procedure of constructing a specific volume temperature curve for each metal or alloy from some temperature well above the melting point or range to room temperature.

The specific volume-temperature curve for liquid metal is constructed from data secured by application of the crucible immersion method, that is, by filling a crucible of known volume with the liquid metal at known temperature. Since the mass of the sample of liquid metal is the same as that of the sample at room temperature, the liquid specific volume may be readily calculated.

Data on the contraction of solid metal were obtained by direct observations of the change in length with change in temperature of a sand-cast bar of metal.

The difference between the specific volume of the liquid metal and of the solid metal at the melting point is the volume shrinkage undergone during solidification. In the case of an alloy, shrinkage during solidification occurs over a range of temperature.

Shrinkage data are given for a number of non-ferrous metals and a cast iron.

EFFECT OF CASTING TEMPERATURE AND OF ADDITIONS OF IRON ON BEARING BRONZE.

While "bronze" is, strictly speaking, a copper-tin alloy, this term now covers a multitude of copper-tin compositions containing a third or even a fourth element. Among the elements that are added, zinc and lead are perhaps the most widely used in bearing bronzes.

A bronze bearing material that has become very popular and which has proved its usefulness in service, is the copper-tin-lead alloy. This alloy known as "leaded bronze" contains hard copper-tin compounds, as hard as some steels, in a softer copper-tin matrix, all of which surrounds particles of uncombined lead. Many successful bearing materials have a structure consisting of hard particles, embedded in a softer matrix. The plasticity of the matrix takes care of small misalignments of the bearing or shaft. The presence of lead, which does not alloy with the copper-tin matrix to any

appreciable extent, assists materially in reducing the friction, renders the bronze more plastic, and aids in the manufacturing and machining of the alloy.

A study has recently been made at the bureau of the effect of casting temperatures and of additions of iron on the properties of bearing bronze material consisting of 80 per cent. copper, 10 per cent. tin, and 10 per cent. lead. In addition to determinations of hardness and microstructure of this leaded bronze, several test methods were employed which simulated certain phases of actual service conditions. These tests included deformation under pounding, and resistance to wear both dry and in the presence of a coolant. As bearings are usually above room temperature while in operation, a considerable number of the above tests were conducted at elevated temperatures.

In the founding of bronzes it is seldom possible to pour all castings at the same temperature because the metal is drawn from the furnace at various temperatures and cools in the ladle during pouring operations. The study of the effect of different casting temperatures thus aids the manufacturer in producing better castings and at the same time helps the consumer by predicting what variations in properties may be expected of bronzes cast at various temperatures. In the study of 80–10–10 bronze cast at temperatures varying from 1,850° to 2,120° F., it was found that as the casting temperature, the grain size, and the resistance to wear were increased, the hardness decreased slightly, and a slight tendency was noted for more even lead distribution. Casting the bronze at about 2,000° F. caused a decrease in the resistance to impact or lowered the toughness as shown by the notch test. The deformation under pounding was a minimum in specimens cast between 1,900° and 2,000° F.

Although iron is not usually added intentionally to leaded bronzes, small amounts often accumulate during the process of manufacture through the use of scrapped bearings, iron stirring rods, and so forth. While the iron content is quite small in most cases, the effect of this element is generally considered harmful. In the reported investigation the iron content, which varied from 0 to 1.0 per cent., was studied to determine its effect upon bearing bronzes. In general, it was

found that iron additions over 0.3 per cent. were detrimental to the bronze investigated. Small additions of iron (up to 0.5 per cent.) decreased the resistance to wear and increased the hardness. When the iron content exceeded 0.3 per cent., segregation of the lead particles occurred, the grain size was reduced, and a decrease was noted in the notch toughness. Additions of iron up to 1.0 per cent. increased the resistance to pounding.

FUNDAMENTAL PROPERTIES OF FIRE CLAY AND FIRE-CLAY PRODUCTS.

In continuation of an extensive investigation of the fundamental properties of fire-clay refractories, a study has been undertaken of the plastic flow, modulus of elasticity, and transverse breaking strength at 1,250° C. of the 17 representative brands of fire-clay brick, a silica brick, and a high alumina brick. Bureau of Standards Research Papers 114 and 194 give data relative to numerous other properties of these fire-brick and the clays from which they were manufactured.

The furnace, which was built especially for this study, requires approximately 4½ hours to heat the 4″ x 6″ x 14″ furnace chamber to 1,250° C. and an additional 3½ hours to obtain the necessary data. It was impossible to obtain data at 1,250° C. of two of the 17 brands of brick, because of their very high plastic deflection. Of the remaining 15 brands the range in modulus of rupture was found to be from 55 to 2,000 lbs./in.^{2}; the range in modulus of elasticity from 75,000 to 520,000 lbs./in.^{2}; and the plastic deflections with a load of approximately 50 lbs./in.^{2} at the center of the specimen, using an 8 inch span, were: minimum, 0.0015 inch, and maximum, 0.044 inch. It was found in general that the silica brick had a comparatively low plastic deflection very similar to the brick high in alumina. Bricks containing from 38 to 48 per cent., approximately, of alumina, have a rather high plastic flow, while those containing less than 38 per cent. have plastic deflections intermediate to the highest and lowest values.

PRODUCTION OF HYDRAULIC LIME FROM DIATOMACEOUS MARL.

Although centuries of experience in Europe have proved hydraulic lime satisfactory and durable, yet in America there are but relatively small quantities of this kind of lime produced. Therefore, when the Kansas Geological Survey requested that a sample of diatomaceous marl from Kansas be examined as a possible source of hydraulic lime, the Bureau agreed. Since a more complete report of the results of this investigation is being written, only a few of the salient factors will be reported at this time.

From a study of heating curves of the diatomaceous marl (which was essentially finely divided calcium carbonate containing about 18 per cent. by weight of diatomaceous silica) it was evident that a reaction was taking place between the lime and the silica at about 300–400° C. lower than in a mixture of chalk containing 18 per cent. of finely ground quartz.

Samples of the diatomaceous marl were then calcined in an electric furnace at definite temperatures for varying lengths of time. A "free lime" determination was then made on each calcined sample and sufficient water was added to react with this CaO to convert it to $Ca(OH)_2$. Briquets were made from this partially hydrated material after it had aged a week in a closed vessel after the addition of the water to permit the CaO to become hydrated. The briquets which contained three parts of standard Ottawa sand and sufficient water to affect the proper consistency were then stored for seven days in the molds in air saturated with water and maintained at 21° C. They were then removed from the molds and stored under water.

It was found that the strength of the briquets was improved if the partially hydrated lime was ground so that about 90 per cent. passed a No. 200 sieve. About 65 per cent. of the dry hydrate passes this sieve if unground. Apparently the best hydraulic lime (as far as early strengths are concerned) resulted from calcining the diatomaceous marl for about an hour at a temperature between 950° and 1,000° C. Briquets made from the ground hydrate developed at the end of 28 days on the average a tensile strength of 230 lbs./in.^{2} and at the end of 90 days about 360 lbs./in.^{2}

Some few tests showed that this hydraulic lime could be used for producing a slightly stronger sand-lime brick than the ordinary high calcium line, an increase in the compressive strength of the particular laboratory brick of from 1,090 to 1,250 lbs./in.^{2} being noted.

HYDRAULIC ENGINEERING AS A PROFESSION.

The following talk was given by H. N. Eaton, acting chief of the bureau's hydraulic laboratory section, under the auspices of the Kiwanis Club of Washington, over radio station WRC of the National Broadcasting Company. It is one of a series of talks on "Vocational Guidance" sponsored by the Kiwanis Club.

Hydraulic engineering is an attractive profession for a limited number of engineers. There are no definite figures available but it appears probable that about 5,000 of the approximately 120,000 engineers in this country are engaged in some form of hydraulic work. However, if you have decided upon an engineering training, and if, in addition, the complicated and elusive processes of flowing water have a fascination for you, then you may well consider whether you may not choose some branch of the field of hydraulics for your life work.

Although I have intimated that the field of hydraulic engineering is limited as regards the number of engineers which it can absorb, nevertheless it offers a great variety of opportunities. If you have a mechanical bent, you may find that the design of hydraulic machinery offers you suitable employment; if you are fond of an out-of-door life, the work of the United States Geological Survey in measuring the flow of streams or the work of the Bureau of Reclamation in designing and erecting irrigation structures may interest you; if the problems involved in the production of power appeal to you, there are hydro-electric power plants to be designed, constructed, and operated.

Finally, if you have an intense desire to learn more of the actual phenomena of flowing water, the engineering colleges offer opportunities for experimental work and for teaching, while several departments of the Federal Government carry on hydraulic experimentation.

The hydraulic engineer has to do both with the art and with the science of hydraulics. American engineers are second to none in their development of the art of hydraulics. However, with some notable exceptions, we have fallen behind Europe during the past quarter century in the development of the science of hydraulics, particularly as regards the methods of solving problems of river hydraulics. We have relied exclusively upon rule-of-thumb methods and upon previous experience when we should have had definite scientific data in order to achieve the best results. Consequently, hydraulic projects costing many millions of dollars have sometimes been only partly successful and in some instances have been total failures.

In a subject so complicated as the flow of fluids, in which only the very simplest problems can be solved analytically and then only when the problems are greatly idealized, it is usually necessary to resort to experiment in order to obtain the answers to our problems. But in order to make an experiment feasible, it is usually necessary to carry it out at a much smaller scale than that of the actual structure. For example, it may be extremely important to design the crest of a dam to pass a certain quantity of water per foot of length when the water level is a given height above the crest, in order to prevent the lowlands upstream from being overflowed in times of flood. Now it is not possible to compute the flow over the crest from theoretical considerations alone. We must rather make an experiment. But neither is it feasible to make the experiment at a scale which is comparable with that of the actual dam. Instead, we must construct a miniature dam geometrically similar to the one which is to be built, with all of the dimensions reduced to perhaps one-fiftieth or one-hundredth of those in the actual structure. Experiments can be made with the model dam and we can multiply the results which we get by the proper factor to make them apply to the full-sized structure.

But, asks the practical engineer upon whose shoulders rests the responsibility for designing the actual dam so that it will do what is expected of it, how can we be sure that the results obtained with the miniature dam can be depended upon to tell us what will happen in the full-sized structure?

The answer to this question is to be found in a branch of the science of mechanics which has been quite generally neglected until recently—in the principles of dynamical similarity. These principles were used by isolated investigators in hydraulics as long ago as half a century. Froude made use of them to develop a method of predicting the resistance of ships based upon tests of small models, a method which has endured to the present day. However, only one or two attempts were made to apply the principles of dynamical similarity to experiments in river hydraulics until about the beginning of the present century. Engels, in Germany, was the leader in this field, and, as a consequence of his success, the method came to be used quite generally in Europe.

American engineers remained unaware of the remarkable progress made abroad in experimental methods in hydraulics until about fifteen years ago, when a prominent American hydraulic engineer, while visiting some of the European hydraulic laboratories, was impressed by the work being done in them and immediately set out to bring it to the attention of the hydraulic engineering profession in the United States. Through the generosity of this man, a number of young hydraulic engineers have been enabled at various times since 1927 to spend a year or more studying in the European hydraulic laboratories and have returned to this country full of enthusiasm for what they have seen. In addition, the publication by the American Society of Mechanical Engineers, of a book entitled "Hydraulic Laboratory Practice," describing the European and American hydraulic laboratories and their work, has brought to the attention of the profession very helpful information as to the new methods which are being used.

As a result there has been given a marked impetus to the construction of hydraulic laboratories in the United States. During the past few years the facilities for hydraulic research in the engineering colleges of this country have been extended greatly, several manufacturers of hydraulic machinery have either built new laboratories or have extended their existing laboratory facilities, the Corps of Engineers of the United States Army has built a laboratory for river experiments at Vicksburg, Mississippi, and a large hydraulic laboratory is

being built at the Bureau of Standards in the Department of Commerce.

I have stressed what has aptly been termed "the new awakening in hydraulic laboratory research" because it is destined to play a very important part in the future design of hydraulic structures and hence must be understood by hydraulic engineers. The Corps of Engineers of the United States Army is already making model tests to aid in the solution of its problems of river control, the Bureau of Reclamation is using this method in studying some of the hydraulic features of the Hoover Dam and other water storage and irrigation structures, and it is becoming customary to subject the hydraulic features of private water power developments to painstaking model investigation before the structures are erected.

It is necessary to emphasize, however, the importance of placing in the hands of properly trained men the task of carrying out model tests and the interpretation of the results, men who have a thorough mastery of the theory of the laws of dynamical similarity, who have the ability to make accurate and painstaking tests, and who are familiar with the practical problems of design and construction, and with the full scale phenomena in Nature.

The hydraulic engineer of the future will require a much better training in the theory of his subject than has been given by American engineering colleges in the past, if he is to remain abreast of his European colleagues. The European engineer is better trained in the fundamentals of his profession than is the American engineer. If he sometimes places a little too much faith in the efficacy of a formula, it is at least better than not knowing that there are any formulas which are applicable. A general knowledge of engineering is very important, and a thorough knowledge of mechanics and a fair command of mathematics are absolutely essential. There is probably no other branch of engineering in which there is so much confusion of thought on the part of engineers as to the phenomena involved as in hydraulics. This is mainly due, in the opinion of the speaker, to the complexity of the phenomena and to the lack of adequate training in the mechanics of fluids.

The engineering colleges do not, in general, offer a definite course in hydraulic engineering leading to a degree. Instead, the requisite studies are included in the curricula in mechanical engineering, civil engineering, and electrical engineering. Under these circumstances, the student who desires to become a hydraulic engineer will be obliged to choose his course by the type of work into which his inclinations tend to lead him. Those whose interest is in hydraulic machinery should obviously choose mechanical engineering as their course of study. Those who are interested in irrigation engineering, the design of dams, or river and harbor improvement should just as obviously select a civil engineering course. The student who is interested in hydro-electric power cannot usually obtain a suitable preparation in either a mechanical engineering or a civil engineering course alone. He will require subjects from both courses and in addition will need training in some aspects of electrical engineering such as power transmission lines and electrical machinery. At present, the only feasible way to obtain specialized engineering training in the field of hydro-electric power would seem to be to take an additional year of college work in engineering or to attend one of the few engineering schools which permits of such specialization. For example, the Massachusetts Institute of Technology offers a hydro-electric power option in its civil engineering course. Many educators oppose specialization of this nature, holding that the student should take a general course in engineering and then specialize after graduation. This is probably a sound view as far as the average student is concerned, but for the unusual student who knows precisely what he wishes to follow for his life work, this specialization would seem to be an advantage, since it can be achieved by omitting certain specialized subjects in other branches of engineering.

The engineer should not be content with being a mere technician. Obviously, he must master the technique of his profession, but this is not sufficient if he is to develop to the fullest possible extent and to exert the influence in his profession and in public affairs which he should. It is impossible to devote the necessary time to cultural studies in an engineering curriculum which is already overcrowded. Conse-

quently, the engineer should supplement his professional training with reading of a cultural nature to broaden his point of view. A more thorough knowledge of foreign languages than is usually obtained in the engineering colleges is not merely desirable, but is of very practical importance. Current hydraulic literature is to be found extensively in the German, French, Italian, Dutch, and Swedish tongues, as well as in English, and, while it can hardly be expected that every hydraulic engineer can spare the time to master all of these languages, he should at least be able to express himself concisely and logically in English. He may have important information to present in a paper, but if he writes in a slipshod manner with a complete disregard of the elementary rules of English composition, much of the effectiveness of his effort will be lost.

A fair idea of the earning power of a hydraulic engineer may be obtained from the report of the committee on the economic status of the engineer, of the American Society of Mechanical Engineers. This report was published in the September, November, and December, 1931 issues of *Mechanical Engineering* under the title: "1930 Earnings of Mechanical Engineers." According to the data presented in this report, if you are an average engineer, you may expect your earnings to be approximately as follows: at 23 to 24 years of age, $2,000 per annum; from 25 to 27 years of age, $2,500 per annum; from 28 to 32 years of age, $3,500 per annum; from 33 to 37 years of age, $4,500 per annum; from 38 to 42 years of age, $5,900 per annum; from 43 to 47 years of age, $6,500 per annum; from 48 to 52 years of age, $7,000 per annum; from 53 to 57 years of age, $7,500 per annum, and from 58 to 62 years of age, $7,500 per annum. The curve is carried only a short distance above the age of 62 years, but shows a slight decrease in earning power. Exceptional men may expect to earn in excess of $25,000 per annum at the peak of their earning power.

Considering the question of sources of employment: hydroelectric power companies require hydraulic engineers for the design and operation of their plants; hydraulic engineers are needed by construction and engineering firms which handle large hydraulic projects; there are opportunities with manu-

facturers of hydraulic turbines, pumps, meters, appliances, etc.; the Corps of Engineers of the United States Army requires, not only officers who are trained as hydraulic engineers, but also civilians, to carry out its immense projects of flood control river regulation, harbor improvement and control of coastal erosion; the United States Bureau of Reclamation has a staff of highly trained hydraulic engineers to design its irrigation works; the United States Geological Survey requires many engineers for the measurement of stream flow; the Bureau of Agricultural Engineering of the United States Department of Agriculture employs hydraulic engineers in research work intended to improve our methods of irrigating and draining agricultural lands, for studying the run-off from agricultural lands and for measuring the flow of irrigation water; and the Bureau of Standards of the United States Department of Commerce is at present erecting a large hydraulic laboratory for experimental research in hydraulics. In addition to the above, some of the States employ hydraulic engineers for the study and development of their water resources and large municipalities require them in connection with their water supply and related problems. The teaching profession also requires a relatively small number of men.

The securing of a diploma should not be regarded as the principal factor which is necessary for success in hydraulic engineering. Technical knowledge is an absolute essential, but given this, it is individual ability which counts most heavily, as in other professions. The brilliant man may have a great advantage over the plodder, but hard, consistent work will go far toward carrying an engineer to the peak of his profession.

THE FRANKLIN INSTITUTE.

ANNUAL MEETING, WEDNESDAY, JANUARY 20, 1932.

The annual meeting of The Franklin Institute was called to order by the President, Mr. Nathan Hayward, at eight-fifteen p.m.

The Chairman called for a reading of the minutes of the previous meeting. The Secretary reported that the minutes had been printed in full in the JOURNAL of the Institute and therefore, he moved that the minutes be approved as printed. No objection was uttered and the minutes were declared approved.

The Chairman then called for the report of the Tellers of the annual election. A statement of the results of the balloting was read by the Secretary, who announced that, the election having been conducted in accordance with our by-laws, the gentlemen whose names appear below are hereby declared elected and shall constitute the officers of The Franklin Institute for the current year, and that the members of the Board of Managers who are also listed are declared elected for the periods indicated:

President (To serve one year)	Nathan Hayward
Vice-Presidents (To serve one year)	Henry Howson
	C. C. Tutwiler
	Walton Forstall
	W. Chattin Wetherill
Treasurer (To serve one year)	Benjamin Franklin
Managers (To serve three years)	Henry B. Allen
	George H. Benzon, Jr.
	G. H. Clamer
	A. W. Goodspeed
	L. H. Kinnard
	Elisha Lee
	Charles Penrose
	G. D. Rosengarten
(To serve two years)	Charles H. Ewing
	Horace P. Liversidge
(To serve one year)	George W. Pepper

On behalf of the Institute, the Secretary then expressed sincere gratitude to the Tellers for the service which they had rendered. The Committee was as follows: Dr. W. M. Stine, Mr. W. N. Jennings, and Mr. J. D. Israel.

The President then resumed the Chair and called upon the Secretary to read a report of the activities of the Institute during the past year. This report appears in this copy of the JOURNAL, immediately following these minutes.

At eight-thirty the President introduced the speaker of the evening, Colonel C. E. MacGuigg, who presented an interesting paper on "Research and Development in Metallurgy."

Colonel MacQuigg's paper dealt with the accomplishment of research in metallurgy and was delightfully illustrated by lantern slides. The reading of the paper was followed by a brisk discussion and the asking of many questions, especially by the younger metallurgists who in large part constituted the audience.

The meeting was adjourned by the President, with a few happy remarks of appreciation of Colonel MacQuigg's lecture and his kindness in coming to the Institute.

HOWARD MCCLENAHAN,
Secretary.

REPORT OF THE FRANKLIN INSTITUTE—FOR THE YEAR 1931.

TO THE PRESIDENT AND MEMBERS OF THE FRANKLIN INSTITUTE:

It is customary and proper to present at this, the annual meeting of the Institute, a report upon the activities of the Institute during the past year. It is a pleasure to be able to follow this procedure at this time, and a source of satisfaction to have much of favorable import to communicate to you.

During the period in question the Institute has been favored by the presence of an unusually large number of interesting and important lecturers, while the meetings which they have addressed have been more largely attended than at any other time within the last six years. Repeatedly, late comers have found standing room only in the lecture hall. The quality of the lecturers has, in all cases, been worthy of the large attendance.

The Christmas Week Lectures on the James Mapes Dodge Foundation, delivered by Dr. Dayton Clarence Miller, were brilliantly successful. The attendance was good and the interest very keen. The experiments shown by Dr. Miller were of very high quality and aroused great enthusiasm.

The Committee on Science and the Arts has done its work admirably during the year and acted finally upon fourteen cases. It still has twenty-two cases under consideration. You will recall, of course, that this Committee is the body which has charge of the award of the medals given by The Franklin Institute. It is made up of sixty members and labors faithfully, giving a great abundance of time to the consideration of the cases before it, and makes awards which arouse the confidence of all those informed of its activities. The annual Medal Day exercises in May, which claim so much public interest, are based entirely upon the labors of this Committee. In May last, for the second time, we were able to have both the Franklin Medalists of the year here in person—Dr. W. R. Whitney of Schenectady, New York, and Sir James H. Jeans of England. Through the endowment provided by Mr. Samuel Insull of Chicago, we were able to have Sir James come to this country to personally receive the Franklin Medal. In addition to our celebration, Sir James gave a number of lectures which were delightfully received throughout the eastern states. His last meeting in this country was a dinner in his honor in New York City, attended by about fifteen hundred people. His presence in the United States was recognized as being due solely to The Franklin Institute.

The JOURNAL of the Institute maintains its high quality and continues to receive favorable comment concerning its make-up and its contents. It carries a

description of the activities of The Franklin Institute to all parts of the civilized world.

For many years Dr. George F. Stradling has been a faithful contributor of current topics to the JOURNAL which have constituted one of the most readable portions of that publication for the general reader. His notes were so lucid and in such interesting style, that the non-technical reader often had difficulties made easy for him which would otherwise have been insurmountable. It is a source of sincere regret that, because of illness, Dr. Stradling has been compelled to forego his valued services to us.

The affairs of the Library are administered with continuing interest in the welfare of the readers and with as much attention to their wants as could possibly be given. The Library is undoubtedly the leading collection in the physical sciences in or about Philadelphia. Its patent literature is unique in this city. Only the lack of funds prevent the Librarian and the Library Committee from keeping the Library in the best possible condition.

The labors of the Bartol Research Foundation are going on with unremitting success. A stream of valuable contributions and results of research is appearing in the technical and scientific journals of the country. The prestige of that laboratory is rising, and with it the reputation of The Franklin Institute.

The membership of the Institute, in spite of the disturbed business conditions,continues to gain. Although there have been a not inconsiderable number of resignations, the additions to membership have exceeded them. With the approaching opening of the Benjamin Memorial and Franklin Institute building on the Parkway, public interest not only in Philadelphia but elsewhere, in The Franklin Institute is growing, with a gratifying increase in the applications for membership.

One is justified in referring to the approaching opening of the Benjamin Franklin Memorial and new Franklin Institute building, for the plans are now complete and are in the hands of the constructors and the scientific staff, for criticism and revision. Some contracts have already been awarded and much of the work has been completed. The subsurface work is practically finished. While certain unavoidable delays have occurred in the preparation of plans, these have all been overcome and all obstacles to the swift erection of the building have been removed.

The scientific staff of the Museum is working vigorously and successfully in the production of exhibits which will be housed on the Parkway. Contributions of historical instruments and machinery are being presented us, scientific masterpieces are being reproduced in our machine shop, and many new and striking instruments for the manifestation of scientific principles are being evolved. As these appear, they are being displayed for the benefit of the public, as a foretaste of the thousands of objects which will be displayed in the new Franklin Institute Museum.

The financial conditions of the Institute and the Memorial project are excellent. In spite of the depression, the collections have been remarkably good. We can now confidently look forward to the completion of the whole structure early in 1933, while it seems reasonably possible to open the planetarium section alone, for the enjoyment of the public, on December 5, 1932, on the third anniversary of the launching of the project for the Benjamin Franklin Memorial.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, January 13, 1932.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JANUARY 13, 1932.

MR. LIONEL F. LEVY *in the Chair.*

The following report was presented for final action:

No. 2949: Franklin Medal.

This report recommended the award of the Franklin Medal to Doctor Phillipp Lenard, of Heidelberg, Germany, "In recognition of a life work devoted to fruitful research in physics, in the course of which he added greatly to scientific knowledge and especially showed that it was possible for cathode rays to exist outside the generating tube and determined the effects produced by such rays; he also discovered the electronic nature of the emission from surfaces upon which ultra-violet light falls, as well as the basic laws of photoelectricity."

GEO. A. HOADLEY,
Secretary.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting Board of Managers, January 20, 1932.)

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DR. H. RUSSELL FISHER, Physician and Teacher, Hahnemann Medical College, Philadelphia, Pa. For mailing: 2124 North Thirty-second Street, Philadelphia, Pa.

DR. STUART KABNICK, Dentist, 1410 Spruce Street, Philadelphia, Pa.

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MR. W. LAURENCE LEPAGE, Executive Aeronautical Engineer, 719 Atlantic Building. For mailing: 400 D Lindley Court, Logan, Philadelphia, Pa.

MR. JOHN MARSHALL, Chemist, Technical Director, Central Technical Laboratory, E. I. duPont de Nemours and Company, 35th Street and Gray's Ferry Road, Philadelphia, Pa.

NON-RESIDENT.

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MR. ALLEN C. STALEY, Mechanical Engineer, Chrysler Corporation, Detroit, Michigan. For mailing: R. F. D., Amberly Road, Birmingham, Michigan.

MR. GEORGE B. STANTON, Junior Engineer, in charge of Standards Section, Research Bureau, Brooklyn Edison Company. For mailing: 505 Twelfth Street, Brooklyn, N. Y.

MR. E. W. UPHAM, Chief Metallurgical Engineer, Chrysler Corporation, Detroit, Michigan. For mailing: 254 Colorado Avenue, Highland Park, Michigan.

MR. HARRY T. WOOLSON, Chief Mechanical Engineer, Chrysler Corporation, Detroit, Michigan.

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DR. EDWARD C. HAINES, 501 East Main Street, Moorestown, N. J.

MR. NATHAN HAYWARD, JR., Dunsten E 44, Harvard University, Cambridge, Mass.

MR. MALCOLM M. HUBBARD, 1597 Main Street, Fourth Floor, Springfield, Mass.

MR. H. H. KNOX, 630 Hot Springs Road, Santa Barbara, California.

PROF. J. C. MCLENNAN, Athenaeum Club, Pall Mall, London, S. W., England.

MR. J. HARTLEY MERRICK, The Wellington, Nineteenth and Walnut Streets, Philadelphia, Pa.

PROF. JOHN P. MORRISSEY, Loyola University, Chicago, Illinois.

MR. BERNARD RHODES, 29 Epwell Road, Erdington, Birmingham, England.

MR. FRANCIS B. VOGDES, 520 Central Avenue, Plainfield, N. J.

NECROLOGY.

Mr. J. Walter Miles, Irwin, Pa.
Mr. Hugh P. Tiemann, Pittsburgh, Pa.

LIBRARY NOTES.

RECENT ADDITIONS.

American Annual of Photography, 1932. Volume XLVI. 1931.

American Institute of Electrical Engineers. Transactions. Volume 50, No. 4. December, 1931.

American Society for Testing Materials. Proceedings of the Thirty-fourth Annual Meeting, 1931. Volume 31, Parts 1–2. Two volumes.

Beilsteins Handbuch der organischen Chemie. Vierte Auflage. Erstes Ergänzungswerk, siebenter und achter Band. 1931.

BENTLEY, W. A., AND W. J. HUMPHREYS. Snow Crystals. First edition. 1931.

BOYER, CHARLES S. Early Forges and Furnaces in New Jersey. 1931.

BURKHARDT, HEINRICH. Theory of Functions of a Complex Variable. Authorized translation from the fourth German edition, with the addition of figures and exercises by S. E. Rasor. Revised. 1913.

CHALMERS, T. W. The Automatic Stabilisation of Ships. 1931.

DANCKWORTT, P. W. Lumineszenz-Analyse im filtrierten ultravioletten Licht. Zweite, erweiterte Auflage. 1929.

FINDLAY, ALEXANDER. Physical Chemistry for Students of Medicine. Second edition. 1931.

FLETCHER, C. NORMAN. The Balancing of Machinery. 1931.

Funk & Wagnalls' New Standard Dictionary of the English Language. 1931.

GAMBLE, SIDNEY GOMPERTZ. A Practical Treatise on Outbreaks of Fire: Being a Systematic Study of their Causes and Means of Prevention. 1926.

GLASSTONE, SAMUEL. Recent Advances in Physical Chemistry. 1931.

GOODMAN, NATHAN G., Editor. The Ingenious Dr. Franklin: Selected Scientific Letters of Benjamin Franklin. 1931.

GRAY, W. S. Reinforced Concrete Reservoirs and Tanks. No date.

HADFIELD, SIR ROBERT A. Faraday and his Metallurgical Researches, with Special Reference to their Bearing on the Development of Alloy Steels. 1931.

HOUGEN, O. A. Industrial Chemical Calculations: the Application of Physico-Chemical Principles and Data to Problems of Industry. 1931.

JACKMAN, D. N. The Chemistry of Laundry Materials. 1931.

KELLY, ERNEST, AND CLARENCE E. CLEMENT. Market Milk. Second edition, rewritten. 1931.

Kidder-Parker Architects' and Builders' Handbook: Data for Architects, Structural Engineers, Contractors, and Draughtsmen. By the late Frank E. Kidder; compiled by a staff of specialists and Harry Parker, Editor-in-Chief. Eighteenth edition. 1931.

KLEIN, G., Editor. Handbuch der Pflanzenanalyse. Erster Band: Allgemeine Methoden der Pflanzenanalyse. 1931.

Klimschs Jahrbuch. Technische Abhandlungen und Berichte über die Neuerungen auf dem Gesamtgebiet der graphischen Künste. Band 25. 1932.

LENZEN, VICTOR F. The Nature of Physical Theory. A Study in Theory of Knowledge. 1931.

LEVENE, P. A., AND LAWRENCE W. BASS. Nucleic Acids. 1931.

LODGE, SIR OLIVER. Atoms and Rays: an Introduction to Modern Views on Atomic Structure and Radiation. 1931.

MANTELL, C. L. Industrial Electrochemistry. First edition. 1931.

MUNRO, JOHN, AND ANDREW JAMIESON, The New "Munro and Jamieson." A Pocket-Book of Electrical Rules and Tables for the Use of Electricians and Engineers. Revised throughout under the editorial oversight of W. R. Cooper and Rollo Appleyard. Twenty-third edition. 1931.

ROGERS, ALLEN, Editor. Industrial Chemistry. A Manual for the Student and Manufacturer. Fifth edition. Two volumes. 1931.

ROHR, MORITZ VON. Joseph Fraunhofers Leben, Leistungen und Wirksamkeit. 1929.

SARTON, GEORGE. Introduction to the History of Science. Volume 2, Parts 1–2. Two volumes. 1931.

SCHUYLER, HAMILTON. The Roeblings: a Century of Engineers, Bridge-Builders and Industrialists. The Story of Three Generations of an Illustrious Family, 1831–1931. 1931.

SHAW, SIR NAPIER. Manual of Meteorology. Volume 4. Meteorological Calculus: Pressure and Wind. 1931.

VAN DEN BROEK, J. A. Elastic Energy. Theory. 1931.

Verein deutscher Ingenieure. Beiträge zur Geschichte der Technik und Industrie. Herausgegeben von Conrad Matschoss. Einundzwanzigster Band. 1931.

Wien-Harms Handbuch der Experimentalphysik. Ergänzungswerk. Band 1. 1931.

BOOK REVIEWS.

JOHANN KEPLER. Edited by Frederick E. Brasch, M.S., Librarian of the Smithsonian Division, the Library of Congress. xii–133 pages, with two plates and three figures, 12mo. Baltimore, The Williams and Wilkins Co., 1931. Price $2.50.

Johann Kepler was a curious mixture of temperaments. He rightly has a place among the very greatest names in the history of astronomy, though he was a good deal of a mystic. He practiced astrology, to him his speculations on the musical notes emitted by the planets, and his theory of the relation of the five regular solids to the planetary orbits, were probably just as important as his three laws of planetary motion, yet it was he who, as the student and scientific heir of Tycho Brahe, developed from his predecessor's observations the empirical laws which later formed the chief foundation on which Newton erected his generalized law of gravitation. Tycho was an observer, par excellence, not a theoretical astronomer. On the other hand, Kepler, though he invented the astronomical telescope with an eyepiece consisting of a convex lens, instead of the concave one used by Galileo, was not so much interested in the observational side of astronomy as in its theory. Thus the two men complemented each other. If they had not happened to meet in Prague on that fateful day in 1600, the year before Tycho's death, probably neither would have the place in history that he now occupies.

Following the commemoration of the tercentenary of his death, held at Würtemberg in 1930, similar exercises were held under the joint auspices of the History of Science Society and Sections A, D and L of the American Association for the Advancement of Science, during the meetings of these bodies in Cleveland, Ohio, in December of that year. The three addresses delivered at this meeting, together with additional material, are now issued as special publication No. 2 of the History of Science Society, under the able editorship of its secretary, Mr. Frederick E. Brasch, of the Library of Congress.

After a brief preface by the editor comes an introduction by Sir Arthur Eddington, who points out that Kepler may be regarded as "the forerunner of the modern theoretical physicist, who is now trying to reduce the atom to order as Kepler reduced the solar system to order." He further reminds us "that in the discovery of the laws of the solar system, as well as of the laws of the atom, an essential step was the emancipation from mechanical models." This step Kepler took when he abandoned the elaborate mechanism of the Ptolemaic theory.

The three addresses deal with the three main phases of Kepler's character. First, Dr. W. Carl Rufus, of the University of Michigan, deals with "Kepler as an Astronomer," giving a very interesting biographical sketch, and account of his astronomical labors. Dr. D. J. Struik, of the Massachusetts Institute of Technology, contributes a chapter on "Kepler as a Mathematician." He describes his contributions to the infinitesimal calculus, to the theory of regular

solids, and the simplification of computations, especially by logarithms, which Kepler was one of the first to use. The third chapter is by Dr. E. H. Johnson, of Kenyon College, on "Kepler and Mysticism." He shows that, mistaken as he may have been, his "association of mystical ideas with physical facts, and his seeking of relations between extra-mundane and human affairs, were sincere attempts to find the unity of the world." Probably the most valuable part of the volume is the last chapter, a bibliography of Kepler's works, prepared by Mr. Brasch. This is the most complete that has yet appeared in English, and its value is enhanced by the illuminating comments on many of the items, the result of the author's scholarly researches. In addition to Kepler's own writings, a number of commentaries are also listed. Unfortunately, one looks in vain through the book for an index.

The History of Science Society deserves commendation for bringing out this excellent tribute to the life and work of so great an astronomer, as the second of their special publications. It is a worthy successor to the first, which was devoted to Sir Isaac Newton, and issued upon the occasion of the bicentenary of his death. However, this reviewer hopes that they will not confine these publications to the commemoration of anniversaries of the death of great scientists. In particular, he hopes that the English translation of the "De Revolutionibus" of Copernicus, now in the custody of the Society, may soon see the light of day.

JAMES STOKLEY.

SIGNALS FROM THE STARS. By George Ellery Hale, LL.D., Sc.D., Ph.D., Honorary Director, the Mount Wilson Observatory. XII–138 pages, with 56 illustrations, 12mo. New York, Chas. Scribner's Sons, 1931. Price $2.00.

Few indeed are the great scientists who can describe their work in so interesting and informative a manner for the lay reader as Dr. Hale. Renowned as the founder and first director of two of the world's greatest observatories, known as one of the greatest authorities on the Sun, recipient of the Franklin Medal and numerous other honors, his scientific position is secure. Though several years ago failing health forced him to retire from the directorship of the Mount Wilson Observatory, he has not been idle. Since then he has developed the spectrohelioscope, the most important of recent inventions for study of the Sun, and has also been mainly responsible for the start of work on a 200 inch telescope for the California Institute of Technology. This little book is the fifth of its kind by the same author, its predecessors being "Ten Years' Work of a Mountain Observatory" (1915), "The New Heavens" (1922), "The Depths of the Universe" (1924) and "Beyond the Milky Way" (1926).

In the four chapters, of which the book consists, are described some of the most important recent developments of astronomy. First is the article (revised) which originally appeared in *Scribner's* on "The Possibilities of Large Telescopes." It was this article, of which reprints were judiciously placed by Dr. Hale, that led to the funds being made available for the 200 inch instrument. The last chapter, latest of the four, describes the first step towards the realization of those possibilities, for it tells of "Building the 200 Inch Telescope." Many problems are yet to be solved before this great engine of research is a reality, but no one acquainted with the work, and with those who have charge of it, doubts their final triumph.

The second chapter, "Exploring the Solar Atmosphere," deals with problems that Dr. Hale has made peculiarly his own. His invention, some years ago, of the spectroheliograph gave astronomers the means of determining the distribution of various elements in the Sun's atmosphere, by photographing it in light of a single wave-length. Now he has produced the spectrohelioscope, which enables the astronomer actually to watch the great flames of hydrogen as they spout from the Sun, or to see the progress of clouds of calcium vapor across its surface. This instrument is doubly of interest to members of The Franklin Institute. Not only will one be included in the equipment of the astronomical section in the new Museum, but also a member, Mr. Gustavus Wynne Cook, now has the first in amateur hands in his observatory at Wynnewood.

In the third chapter, "Signals from the Sun," the author tells more of the results of the exploration of its atmosphere, and calls attention to the desirability of a complete chain of solar observatories encircling the Earth, so that the Sun might never be unobserved. Already this hope has begun to be realized.

Whether or not one is particularly interested in astronomy, this book should be a delight to anyone who follows scientific developments, for here are answered many questions concerning the latest ideas about the Universe, and how the answers are being found. The numerous illustrations, including several reproductions of drawings by Russell W. Porter, as well as some particularly fine astronomical photographs, are deserving of special mention. The only adverse criticism this reviewer has to make is that the book would have been much more useful if the publishers had provided an index.

JAMES STOKLEY.

ATM—ARCHIV FUR TECHNISCHES MESSEN. Ein sammelwerk für die gesamte Messtechnik, herausgegeben von Prof. Dr. Ing. Georg Keinath. Lieferung 1, 2, 3, July, August, September 1931. München, R. Oldenbourg, 1931. Price per issue, 1.50 marks.

As may be gathered from the title, this publication will be devoted to all problems relating to measuring and measuring devices, whether operated by mechanical or electrical means. Contributions by specialists from all parts of the world will be published from time to time. All the important articles will include references to recent literature. Supplementary issues will be included, when necessary, giving accounts of newly invented instruments and accessories, their use and operation.

The pages of each issue are perforated and classified according to a decimal system which will enable each subscriber to classify the several kinds of contributions according to their specific subjects. There are three main divisions in this classification: Procedures in measurement, with about fifty sub-divisions; Instruments, apparatus, with sixty sub-divisions; Accessories, with forty-one sub-divisions.

In a short time each reader will have at his disposal a complete compendium of all data relating to this important branch of technology. The editor of this new publication is already well known as the author of a work on electrical measuring instruments, which has passed through several editions.

R.

CANADA, DEPARTMENT OF MINES, MINES BRANCH, CHRYSOTILE ASBESTOS IN CANADA. By James Gordon Ross, 146 pages, illustrations, plates, maps, tables, 8vo. Ottawa, King's Printer, 1931. Copies may be purchased from the Director, Mines Branch, Department of Mines, Ottawa, Canada. Price, twenty cents.

The third edition of a monograph on asbestos, prepared by Captain James G. Ross, Mining Engineer, Montreal, has just been issued. The new volume, through seven chapters, describes all phases of the industry from mining to industrial fabrication. The report is well illustrated with many illuminating half-tones. Descriptions of the principal asbestos mining operations in Canada are supplemented by chapters dealing with the preparation of asbestos for marketing, trade requirements, and the application of asbestos fibres in many industries.

Asbestos, a mineral product renowned for its insulating and fire-resisting qualities, has been one of the principal mineral products of Canada for four decades. Known to the ancients for centuries, the first discoveries of importance in America were made near Thetford, Quebec, in 1862. An abundant supply stimulated search for new and varied uses, and as the demand for fibres for industrial uses grew the mining industry expanded until at the beginning of this century it has become one of our most important industries.

In 1904 the Mines Branch, at Ottawa, arranged for the preparation of an exhaustive monograph on Canadian Chrysotile Asbestos and its uses by the late Fritz Cirkel. Five years later a second edition became necessary, the industry having notably increased in importance in the meantime. Intervened the war and early post-war years with increasing demand for the products, especially from the automobile industry and from the producers of spun and woven asbestos fireproof products, a demand which lasted to the beginning of the present industrial depression. This third edition contains much new data.

R.

THE STRUCTURE OF CRYSTALS. By Ralph W. G. Wyckoff, American Chemical Society Monograph Series, Second Edition. 497 pages, ill., 8vo. New York, The Chemical Catalog Company, Inc., 1931. Price $7.50.

So rapid has been the increase in knowledge on the structure of crystals within the past few years, the author has found good justification for issuance of a second edition. This increase, coupled with a constant expansion of the field, has made it necessary to limit further the scope of the single volume. This enforced limitation is compensated by a more complete and thorough treatment of the subject matter embraced by its covers.

The book consists of two parts: Part I, Methods of Crystal Analysis and Part II, Results of Crystal Analysis. Part I is a broad and not too detailed discussion of crystal analysis methods. It very appropriately begins with an introduction on the gradual maturation of ideas and thought on crystal structure. This is followed by a chapter on crystal symmetry following which the author discusses some of the properties of x-rays and their interaction with crystals. After these more or less introductory chapters the author comes to the more serious subject of describing just how the x-ray spectroscopic photographs are made and a careful description of how, when once obtained, they shall be interpreted. This includes both the single crystal and powder methods. The subject of x-ray spec-

trometry by means of crystals has not been slighted and the last chapter of Part I, probably the most valuable, outlines a logical system of approaches for the determination of crystal structure.

Part II is more of an encyclopædic nature in that successive chapters serve to classify and illustrate the structures of the elements and their compounds in increasing order of complexity. Sufficient work has been done on the hydrates ammoniates and miscellaneous inorganic compounds to merit a separate chapter. The structures of silicates have claimed the attention of a good many workers in the field and much headway made despite the hypercomplexity of these natural substances. Studying the structures of organic compounds is comparatively difficult, but, of all the results, probably the most eagerly awaited. In these studies is seen the possible verification of the many hypotheses made regarding the atomic structure of organic compounds.

The book is furnished with a bibliography of crystal structure data chronologically arranged and should prove a valuable source of reference to workers in the field.

T. K. CLEVELAND.

PUBLICATIONS RECEIVED.

Applied Gyrodynamics, for Students, Engineers and Users of Gyroscopic Apparatus, by Ervin S. Ferry, 277 pages, illustrations, 8vo. New York, John Wiley & Sons, Inc., 1932. Price $4.00.

Theoretical Physics, by W. Wilson, Vol. 1, Mechanics and Heat, Newton-Carnot, 332 pages, diagrams, 8vo. New York, E. P. Dutton & Company, Inc. Price $6.00.

The Queen of the Sciences, by E. T. Bell, 138 pages, 12mo. Baltimore, The Williams & Wilkins Company, 1931. Price $1.00.

France, A Regional and Economic Geography, by H. Ormsby, 515 pages, diagrams, maps, 8vo. New York, E. P. Dutton & Company, Inc. Price $6.50.

The Mississippi River and Valley, Bibliography, 125 pages, 8vo. Fort Humphreys, The Engineer School, 1931. Price ten cents.

Canada, Department of Mines, Mines Branch. Chrysotile Asbestos in Canada, by Mames Gordon Ross, 146 pages, illustrations, plates, maps, tables, 8vo. Ottawa, King's Printer, 1931. Price twenty-five cents.

Pratt Institute, School of Science & Technology, Michael Faraday 1791–1867, a selected list of books and periodical literature, 13 pages, 8vo. Brooklyn, Pratt Institute, 1931.

National Advisory Committee for Aeronautics, Seventeenth Annual Report, 1931, 62 pages, plates, quarto. Washington, Government Printing Office, 1931. Price twenty cents. No. 402, The Effectiveness of a Double-Stem Injection Valve in Controlling Combustion in a Compression-Ignition Engine—by J. A. Spanogle and E. G. Whitney, Langley Memorial Aeronautical Laboratory, 19 pages, diagrams, quarto. Washington, Committee, 1931. No. 403, The Interference Effects on an Airfoil of a Flat Plate at Mid-Span Position, by Kenneth E. Ward, Langley Memorial Aeronautical Laboratory, 16 pages, diagrams, quarto. Washington, Committee, 1931.

U. S. Bureau of Standards, Research Paper No. 368, The Porosity of Electroplated Chromium Coatings, by W. Blum, W. P. Barrows, A. Brenner, 14 pages, plates, diagrams, 8vo. Washington, Superintendent of Documents. Price ten cents.

Bell Telephone Laboratories, Monographs: No. B-609, A Rapid Record Oscillograph, by A. M. Curtis and I. E. Cole, 7 pages, diagrams, 8vo. No. B-610, High Frequency Atmospheric Noise, by R. K. Potter, 35 pages, tables, diagrams, 8vo. New York, 1931.

U. S. Department of Agriculture, Farmers' Bulletin No. 1682, Usefulness of Birds on the Farm, 14 pages, illustrations, maps, 8vo. Washington, Government Printing Office, 1931.

THE FRANKLIN INSTITUTE

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AWARDS BY THE INSTITUTE

The following awards are made by The Franklin Institute:

The Franklin Medal (Gold Medal).—This medal is awarded annually from the Franklin Medal Fund, founded January 1, 1914, by Samuel Insull, Esq., to those workers in physical science or technology, without regard to country, whose efforts, in the opinion of the Institute, acting through its Committee on Science and the Arts, have done most to advance a knowledge of physical science or its applications.

The Elliott Cresson Medal (Gold Medal).—This medal is awarded for discovery or original research, adding to the sum of human knowledge, irrespective of commercial value; leading and practical utilizations of discovery; and invention, methods or products embodying substantial elements of leadership in their respective classes, or unusual skill or perfection in workmanship.

The Howard N. Potts Medal (Gold Medal).—This medal is awarded for distinguished work in science or the arts; important development of previous basic discoveries; inventions or products of superior excellence or utilizing important principles.

The George R. Henderson Medal (Gold Medal).—This medal is to be awarded for meritorious inventions or discoveries in the field of railway engineering.

The John Price Wetherill Medal (Silver Medal).—This medal is awarded for discovery or invention in the physical sciences or for new and important combinations of principles or methods already known.

The Edward Longstreth Medal (Silver Medal).—This medal is awarded for inventions of high order and for particularly meritorious improvements and developments in machines and mechanical processes. In the event of an accumulation of the fund for medals beyond the sum of one hundred dollars, it is competent for the Committee on Science and the Arts to offer from such surplus a money premium for some special work on any mechanical or scientific subject that is considered of sufficient importance.

The Louis E. Levy Medal (Gold Medal).—This medal is awarded to the author of a paper of especial merit, published in the JOURNAL OF THE FRANKLIN INSTITUTE, preference being given to one describing the author's experimental and theoretical researches in a subject of fundamental importance.

The Walton Clark Medal (Gold Medal).—This medal is awarded to the "author of the most notable advance in knowledge or improvement in apparatus, or in method concerning the science or the art of gas manufacture or distribution or utilization in the production of illumination, or of heat, or of power."

The Certificate of Merit.—A Certificate of Merit is awarded to persons adjudged worthy thereof for meritorious inventions, discoveries or improvements in physical processes or devices.

The Boyden Premium.—Uriah A. Boyden, Esq., of Boston, Mass., has deposited with The Franklin Institute the sum of one thousand dollars, to be awarded as premium to "any resident of North America who shall determine by experiment whether all rays of light and other physical rays are or are not transmitted with the same velocity."

For further information relating to these awards apply to the Secretary of the Institute.